

DISPLACEMENT, VELOCITY AND ACCELERATION IN SHM

Displacement $x = A\sin(\omega t + \varphi)$

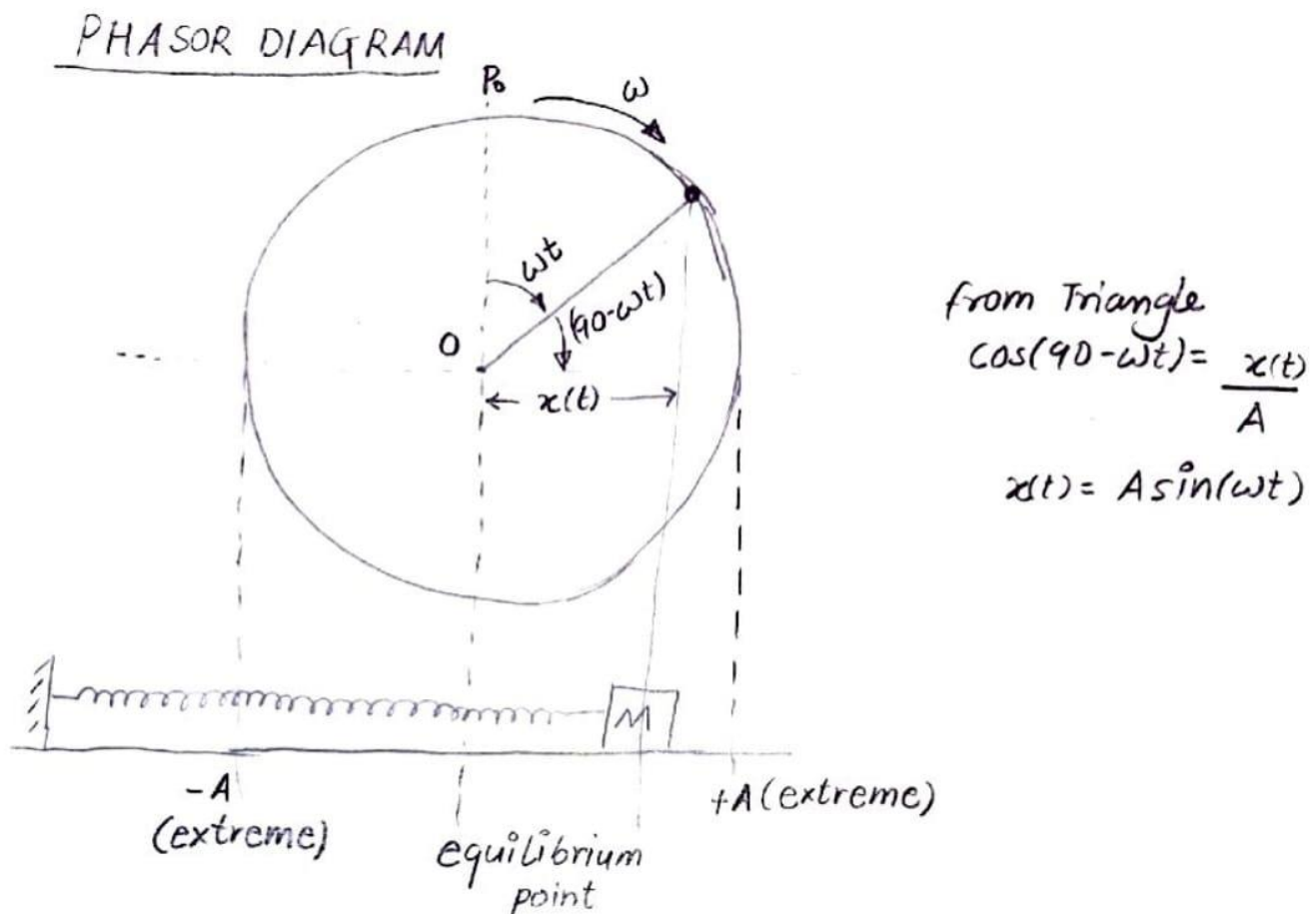
Velocity $v = A\omega\cos(\omega t + \varphi) = A\omega\sin\left(\omega t + \varphi + \frac{\pi}{2}\right)$ or finally $v = \omega\sqrt{A^2 - x^2}$

Acceleration $a = -\omega^2 A\sin(\omega t + \varphi) = \omega^2 A\sin(\omega t + \varphi + \pi)$ or finally $a = -\omega^2 x$

Time , t	0 (Mean Position)	T/4 (Extreme Position)	T/2 (Mean Position)	3T/4 (Extreme Position)	T (Mean Position)
Displacement, x	0	A	0	-A	0
Velocity, v	A ω	0	-A ω	0	A ω
Acceleration, a	0	- $\omega^2 A$	0	$\omega^2 A$	0

Graph of Velocity vs Displacement will be Elliptical.

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION



ENERGY OF SHM

KINETIC ENERGY (KE)

- $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$ as $\omega^2 = \frac{k}{m}$
- $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 \cos^2(\omega t + \phi)$ as $v = A\omega \cos(\omega t + \phi)$
- $KE_{\max} = \frac{1}{2}kA^2$ when v is max - at mean position.

* frequency of KE = 2 (frequency of SHM)

i.e. Graph of KE completes two cycle in one Time Period of SHM.

POTENTIAL ENERGY (PE)

$$\frac{1}{2}kx^2 = \frac{1}{2}kA^2 \sin^2(\omega t + \phi)$$

$$PE_{\max} = \frac{1}{2}kA^2 \text{ at extreme position.}$$

- Same frequency as KE.

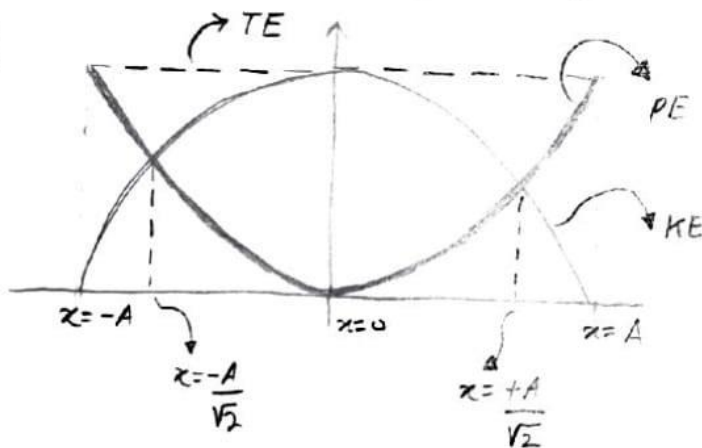
TOTAL MECHANICAL ENERGY (ME)

$$ME = KE + PE$$

$$= \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$$

constant throughout the motion.

GRAPH



The Potential Energy & Kinetic Energy are equal to $x = \pm \frac{A}{\sqrt{2}}$