Q) If $x = 2 + 2^{1/3} + 2^{2/3}$, then $x^3 - 6x^2 + 6x = ?$

Sol:

given $x = 2 + 2^{1/3} + 2^{2/3}$

$$x - 2 = 2^{1/3} + 2^{2/3}$$

cubing on both sides

$$\Rightarrow (x-2)^3 = \left(2^{1/3} + 2^{2/3}\right)^3$$

$$\Rightarrow x^3 - 8 - 6x^2 + 12x = 2 + 4 + 3 \cdot 2^{2/3} \cdot 2^{2/3} + 3 \cdot 2^{1/3} \cdot 2^{4/3}$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 3\left(2^{4/3} + 2^{2/3}\right)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 3 \cdot 2\left(2^{1/3} + 2^{2/3}\right)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6(x-2)$$

$$\Rightarrow x^3 - 6x^2 + 12x - 8 = 6 + 6x - 12$$

$$\Rightarrow x^3 - 6x^2 + 6x - 2 = 0$$

$$\Rightarrow \boxed{x^3 - 6x + 6x = 2}$$ //