2. Let $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)}$; $0<x<2$, m and n are integers, $m\neq 0,\ n>0$, and let p be the left hand derivative of $|x-1|$ at $x=1$. If $\lim\limits_{x\to 1^+} g(x)=p$, then *(2008)*

(a) $n=1, m=1$ (b) $n=1, m=-1$

(c) $n=2, m=2$ (d) $n>2, m=n$

Solution: -

2. (c) As per question,

$p=$ left hand derivative of $|x-1|$ at $x=1 \Rightarrow p=-1$

Also $\lim\limits_{x\to 1+} g(x)=p$

Where $g(x)=\dfrac{(x-1)^n}{\log\cos^m(x-1)}$, $0<x<2$,

m, n are integers, $m\neq 0, n>0$

$\therefore$ we get,

$$\lim_{x\to 1^+}\frac{(x-1)^n}{\log\cos^m(x-1)}=-1 \Rightarrow \lim_{h\to 0}\frac{h^n}{\log\cos^m h}=-1$$

$$\Rightarrow \lim_{h\to 0}\frac{h^n}{m(\log\cos h)}=-1 \text{ [Using L' Hospital's rule]}$$

$$\Rightarrow \lim_{h\to 0}\frac{n h^{n-1}\cos h}{m(-\sin h)}=-1 \text{ [Using L' Hospital's rule]}$$

$$\Rightarrow \lim_{h\to 0}\frac{n h^{n-2}\cos h}{m\left(\frac{\sin h}{h}\right)}=1 \Rightarrow n=2 \text{ and } m=2$$