**4.** Let $m$ and $n$ be two positive integers greater than 1. If

$$\lim_{\alpha\to 0}\left(\frac{e^{\cos(\alpha^n)}-e}{\alpha^m}\right)=-\left(\frac{e}{2}\right)$$ then the value of $\frac{m}{n}$ is

***(JEE Adv. 2015)***

**Solution: -**

**4. (2)** $$\lim_{\alpha\to 0}\frac{e^{\cos\alpha^n}-e}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow \lim_{\alpha\to 0}\frac{e\left[e^{\cos\alpha^n-1}-1\right]}{\cos\alpha^n-1}\times\frac{\cos\alpha^n-1}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow e\lim_{\alpha\to 0}\frac{-2\sin^2\frac{\alpha^n}{2}}{\left(\frac{\alpha^n}{2}\right)^2}\times\frac{\left(\frac{\alpha^n}{2}\right)^2}{\alpha^m}=\frac{-e}{2}$$

$$\Rightarrow \frac{-e}{2}\alpha^{2n-m}=\frac{-e}{2} \text{ or } \alpha^{2n-m}=1$$

$$\Rightarrow 2n-m=0\Rightarrow\frac{m}{n}=2$$