5. Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{\alpha x - \sin x} = 1$. Then $6(\alpha + \beta)$ equals. ***(JEE Adv. 2016)***

Solution: -

5. **(7)** $\lim_{x \to 0} \frac{x^2 \sin \beta x}{\alpha x - \sin x} = 1$

$$\Rightarrow \lim_{x \to 0} \frac{x^3 \beta}{\alpha x - \sin x} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{x^3 \beta}{\alpha x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots.\infty\right)} = 1$$

$$\Rightarrow \lim_{x \to 0} \frac{x^3 \beta}{(\alpha - 1)x + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots.\infty} = 1$$

For above to be possible, we should have

$\alpha - 1 = 0$ and $\beta = \frac{1}{3!}$

$\Rightarrow \alpha = 1$ and $\beta = \frac{1}{6}$

$$\therefore 6(\alpha + \beta) = 6\left(1 + \frac{1}{6}\right) = 7$$