Determinants - Class XII

Related Questions with Solutions

Questions

Quetion: 01

If $D_r = \begin{vmatrix} r & r^3 & 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix}$, then $\sum_{r=0}^{n} D_r =$

A. n
B. 2n
C. 1
D. 0

Quetion: 02

If $A+B+C=\pi$, then value of $\begin{vmatrix} \sin(A+B+C) & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(A+B) & -\tan A & 0 \end{vmatrix}$ is

A. 0
B. 1
C. 2 sinA tanA cosC
D. 2 sinA sinBsinC

Quetion: 03

If $ax^4+bx^3+cx^2+dx+e = \begin{vmatrix} 2x & x-1 & x+1 \\ x+1 & x^2-x & x-1 \\ x-1 & x+1 & 3x \end{vmatrix}$, then the value of e , is

A. 0
B. -2
C. 3
D. 2

Quetion: 04

If $A(x_1, y_1), B(x_2, y_2)$ and $C(x_3, y_3)$ are the vertices of an equilateral triangle whose each side is equal to a, then $\begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2$ is equal to

A. $2a^2$
B. $2a^4$
C. $3a^2$
D. $3a^4$

Quetion: 05

Three distinct points $P(3u^2, 2u^3)$; $Q(3v^2, 2v^3)$ and $R(3w^2, 2w^3)$ are collinear then uv + vw + wu =

A. 0
B. -1
C. 1
D. 3

Solutions

Solution: 01

$$\sum_{r=0}^{n} D_r = \begin{vmatrix} \sum_{r=0}^{n} r & \sum_{r=0}^{n} r^3 & \sum_{r=0}^{n} 2 \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = \begin{vmatrix} \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \\ n & n^3 & 2n \\ \frac{n(n+1)}{2} & \left(\frac{n(n+1)}{2}\right)^2 & 2(n+1) \end{vmatrix} = 0$$

Solution: 02

$$\Delta = \begin{vmatrix} \sin \pi & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ \cos(\pi - C) & -\tan A & 0 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 0 & \sin B & \cos C \\ -\sin B & 0 & \tan A \\ -\cos C & -\tan A & 0 \end{vmatrix} = 0$$ [Skew symmetric determinant of odd order]

Solution: 03

Putting $x = 0$, we have

$$\begin{vmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & 0 \end{vmatrix} \quad [C_2 \to C_2 + C_3]$$

= 1 – 1 = 0

Solution: 04

Step I : Find the area of triangle using determinant

Let area of ΔABC be Δ

Then, $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$\Rightarrow 2\Delta = \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} \Rightarrow 4\Delta = \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}$$

Step II : Find the area of equilateral triangle whose side is a

$$\therefore \quad \Delta = \frac{\sqrt{3}}{4} a^2$$

$$\Rightarrow \quad 4\Delta = \sqrt{3}a^2$$

$$\Rightarrow 16\Delta^2 = 3a^4$$

$$\therefore \begin{vmatrix} x_1 & y_1 & 2 \\ x_2 & y_2 & 2 \\ x_3 & y_3 & 2 \end{vmatrix}^2 = 3a^4$$

Solution: 05

$$\begin{vmatrix} 3u^2 & 2u^3 & 1 \\ 3v^2 & 2v^3 & 1 \\ 3w^2 & 2w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$ and $R_2 \to R_2 - R_3$

$$\Rightarrow \begin{vmatrix} u^2 - v^2 & u^3 - v^3 & 0 \\ v^2 - w^2 & v^3 - w^3 & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} u+v & u^2+v^2+vu & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2$

$$\Rightarrow \begin{vmatrix} u - w & (u^2 - w^2) + v(u - w) & 0 \\ v + w & v^2 + w^2 + vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0 \quad \Rightarrow \begin{vmatrix} 1 & u+w+v & 0 \\ v+w & v^2+w^2+vw & 0 \\ w^2 & w^3 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \left(v^2 + w^2 + vw\right) - (v + w)[(v + w) + u] = 0$$

$$\Rightarrow v^2 + w^2 + vw = (v + w)^2 + u(v + w)$$

$\Rightarrow$ uv + vw + wu = 0 Ans.

Correct Options

Answer:01
Correct Options: D
Answer:02
Correct Options: A
Answer:03
Correct Options: A
Answer:04
Correct Options: D
Answer:05
Correct Options: A