Practice Questions

Q1.

Page-77

6. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

Sol. We have, $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$

[Applying $R_1 \to R_1 + R_2 + R_3$]

$$= \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Taking $(a+b+c)$ common from the first row]

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

[Applying $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$]

$$= (a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ 0 & -(a+b+c) & 2b \\ a+b+c & a+b+c & c-a-b \end{vmatrix}$$

Expanding along R_1,

$$= (a+b+c)\,[1 \times 0 + (a+b+c)^2] = (a+b+c)^3$$

Q2.

Page-85

58. The maximum value of $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$ is $\frac{1}{2}$.

Sol. True

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

[Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$]

$$= \begin{vmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & 0 \\ 0 & 0 & \cos\theta \end{vmatrix} = \cos\theta \cdot \sin\theta = \frac{1}{2}\sin 2\theta$$

Therefore, maximum value of Δ is $\frac{1}{2}$, when $\sin 2\theta = 1$.

Q3.

14. If $a_1, a_2, a_3, \ldots, a_r$ are in G.P., then prove that the determinant $\begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$ is independent of r.

Sol. We know that,

$$a_{r+1} = AR^{(r+1)-1} = AR^r;$$

where a_r = rth term of G.P.,

A = First term of G.P.

and R = Common ratio of G.P.

$$\therefore \quad \begin{vmatrix} a_{r+1} & a_{r+5} & a_{r+9} \\ a_{r+7} & a_{r+11} & a_{r+15} \\ a_{r+11} & a_{r+17} & a_{r+21} \end{vmatrix}$$

$$= \begin{vmatrix} AR^r & AR^{r+4} & AR^{r+8} \\ AR^{r+6} & AR^{r+10} & AR^{r+14} \\ AR^{r+10} & AR^{r+16} & AR^{r+20} \end{vmatrix}$$

[Taking AR^r, AR^{r+6} and AR^{r+10} common from R_1, R_2 and R_3, respectively]

$$= AR^r \cdot AR^{r+6} \cdot AR^{r+10} \begin{vmatrix} 1 & AR^4 & AR^8 \\ 1 & AR^4 & AR^8 \\ 1 & AR^6 & AR^{10} \end{vmatrix}$$

$= 0$ [As R_1 and R_2 are identical]

Q4.

56. If the determinant $\begin{vmatrix} x+a & p+u & l+f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$ splits into exactly K determinants of order 3, each element of which contains only one term, then the value of K is 8.

Sol. True

Since, $\begin{vmatrix} x+a & p+u & l+f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$

[Splitting w.r.t. R_1]

$$= \begin{vmatrix} x & p & l \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix} + \begin{vmatrix} a & u & f \\ y+b & q+v & m+g \\ z+c & r+w & n+h \end{vmatrix}$$

[Splitting w.r.t. R_2 in each determinant]

$$= \begin{vmatrix} x & p & l \\ y & q & m \\ z+c & r+m & n+h \end{vmatrix} + \begin{vmatrix} x & p & l \\ b & v & g \\ z+c & r+w & n+h \end{vmatrix}$$

$$+ \begin{vmatrix} a & u & f \\ y & q & m \\ z+c & r+w & n+h \end{vmatrix} + \begin{vmatrix} a & u & f \\ b & v & g \\ z+c & r+w & n+h \end{vmatrix}$$

Similarly, we can split these 4 determinants in 8 determinants by splitting each one in two determinants further. So, given statement is true.