Determinants - Class XII

Related Questions with Solutions

Questions

Quetion: 01

The system of linear equations
$\lambda x+2y+2z=5$

$2\lambda x+3y+5z=8$

$4x+\lambda y+6z=10$ has:
A. infinitely many solutions when $\lambda=2$
B. no solution when $\lambda=2$
C. No solution when $\lambda=8$
D. a unique solution when $\lambda=-8$

Quetion: 02

The value of λ such that the system
$x-2y+z=-4, 2x-y+2z=2, x+y+\lambda z=4$ has no solution is
A. 0
B. 1
C. -1
D. 3

Quetion: 03

If the system of linear equations
$x+ky+3z=0$
$3x+ky-2z=0$
$2x+4y-3z=0$
has a non-zero solutions (x, y, z), then $\frac{xz}{y^2}$ is equal to

A. 30
B. -10
C. 10
D. -30

Quetion: 04

If the system of linear equations
$x+2ay+az=0$

$x+3by+bz=0$
$x+4cy+cz=0$ has a non-trivial solution and a, b and $c>0$ then the minimum value of $\left(\frac{a^3+c^3}{b^3}\right)$ is
A. 1
B. 0
C. 2
D. 3

Quetion: 05

If the system of equations $x+\lambda y+1=0, \lambda x+y+1=0 \& x+y+\lambda=0.$ is consistent then the value(s) of λ is(are) -
A. 1
B. -1
C. 2
D. -2

Quetion: 06

The values of θ, λ for which the following equations
$\sin\theta x-\cos\theta y+(\lambda+1)z=0; \cos\theta x+\sin\theta y-\lambda z=0; \lambda x+(\lambda+1)y+\cos\theta z=0$
have non trivial solution, is
A. $2n\pi \forall n \in I$

B. $n\pi \forall n \in I$
C. $(2n+1)\frac{\pi}{2} \forall n \in I$
D. No value possible

Solutions

Solution: 01

$$\Delta = \begin{bmatrix} \lambda & 2 & 2 \\ 2\lambda & 3 & 5 \\ 4 & \lambda & 6 \end{bmatrix}$$

$|\Delta| = \lambda(18-5\lambda) - 2(12\lambda - 20) + 2\left(2\lambda^2 - 12\right)$
$= 18\lambda - 5\lambda^2 - 24\lambda + 40 + 4\lambda^2 - 24$
$= -\lambda^2 - 6\lambda + 16$
$If\ |\Delta| = 0 \Rightarrow \lambda^2 + 6\lambda - 16 = 0$
$(\lambda+8)(\lambda-2) = 0$
$\lambda = -8\ \ or\ \lambda = 2$

$$If\ \lambda = 2 : \Delta_x = \begin{bmatrix} 5 & 2 & 2 \\ 8 & 3 & 5 \\ 10 & 2 & 6 \end{bmatrix}$$

$|\Delta_x| = 5(18-10) - 2(48-50) + 2(16-30)$
$= 40 + 4 - 28 \neq 0$

So no solution for
$\lambda = 2$

Solution: 02

Here, $\Delta = \begin{vmatrix} 1 & -2 & 1 \\ 2 & -1 & 2 \\ 1 & 1 & \lambda \end{vmatrix} = 1(-\lambda-2) + 2(2\lambda-2) + 1(2+1)$ = 0 gives $\lambda = 1$

and $\Delta_x = \begin{vmatrix} -4 & -2 & 1 \\ 2 & -1 & 2 \\ 4 & 1 & \lambda \end{vmatrix}$

$\Delta_x = -4(-\lambda-2) + 2(2\lambda-8) + 1(2+4) \neq 0$
Now, for no solution, we must have $\Delta = 0$.
Clearly for $\lambda = 1, \Delta = 0$ and $\Delta_x \neq 0$
Hence, system of equations has no solution for $\lambda = 1$.

Solution: 03

For non-zero solutions, we must have

$$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$$

$\Rightarrow \quad 1(-3k+8) - k(-9+4) + 3(12-2k) = 0$
which gives $k = 11$
Now, the system of equations become
$x + 11y + 3z = 0$ ...[i]
$3x + 11y - 2z = 0$ ...[ii]
$2x + 4y - 3z = 0$ ...[iii]
The equation [i] and [iii] gives
$3x + 15y = 0$ *i.e.* $x = -5y$
Putting $x = -5y$ in [i], we get
$-5y + 11y + 3z = 0 \Rightarrow z = -2y$

Now $\frac{xz}{y^2}=\frac{(-5y)(-2y)}{y^2}=10$

Solution: 04

$\Delta=0 \Rightarrow \mathrm{b}=\frac{2\mathrm{ac}}{\mathrm{a}+\mathrm{c}}$

Now, $t=\frac{a^3+c^3}{b^3}=\frac{(a^3+c^3)}{8a^3c^3}\times(a+c)^3$

$=\frac{(a+c)^4(a^2-ac+c^2)}{8a^3c^3}$ ....[i]

Now, $(a-c)^2=a^2-2ac+c^2\geq 0$

$\Rightarrow a^2-ac+c^2\geq ac$ ....[ii]

Also, $a+c\geq 2\sqrt{ac}\Rightarrow (a+c)^4\geq 16a^2c^2$ ...[iii]

Multiplying [II] and [III], we get

$(a+c)^4(a^2-ac+c^2)\geq 16a^3c^3$

$\Rightarrow \frac{(a+c)^4(a^2-ac+c^2)}{8a^3c^3}\geq 2 \quad \Rightarrow t\geq 2$

Solution: 05

For consistency of the given system of equations

$$D=\begin{vmatrix} 1 & \lambda & 1 \\ \lambda & 1 & 1 \\ 1 & 1 & \lambda \end{vmatrix}=0$$

$\lambda^3-3\lambda+2=0\Rightarrow(\lambda-1)^2(\lambda+2)=0\Rightarrow\lambda=1$ or $\lambda=-2$

Solution: 06

For non trivial solution $\begin{vmatrix} \sin\theta & -\cos\theta & \lambda+1 \\ \cos\theta & \sin\theta & -\lambda \\ \lambda & \lambda+1 & \cos\theta \end{vmatrix}=0;$ this gives

$2\cos\theta\left(\lambda^2+\lambda+1\right)=0$

Correct Options

Answer:01
Correct Options: B
Answer:02
Correct Options: B
Answer:03
Correct Options: C
Answer:04
Correct Options: C
Answer:05
Correct Options: A, D
Answer:06
Correct Options: C