4 JEE Main 2021 (Online) 17th March Morning Shift

MCQ (Single Correct Answer)

The system of equations kx + y + z = 1, x + ky + z = k and x + y + zk = k^2 has no solution if k is equal to :

A 0

B -1

C -2

D 1

Explanation

$$D = \begin{vmatrix} k & 1 & 1 \\ 1 & k & 1 \\ 1 & 1 & k \end{vmatrix} = 0$$

$$\Rightarrow k(k^2 - 1) - (k - 1) + (1 - k) = 0$$

$$\Rightarrow (k - 1)(k^2 + k - 1 - 1) = 0$$

$$\Rightarrow (k - 1)(k^2 + k - 2) = 0$$

$$\Rightarrow (k - 1)(k - 1)(k + 2) = 0$$

$$\Rightarrow k = 1, k = -2$$

for k = 1 equation identical so k = -2 for no solution.

4 JEE Main 2021 (Online) 26th August Morning Shift

MCQ (Single Correct Answer)

Let $\theta \in (0, \frac{\pi}{2})$. If the system of linear equations

$(1+\cos^2\theta)x + \sin^2\theta y + 4\sin 3\theta z = 0$

$\cos^2\theta x + (1+\sin^2\theta)y + 4\sin 3\theta z = 0$

$\cos^2\theta x + \sin^2\theta y + (1+4\sin 3\theta)z = 0$

has a non-trivial solution, then the value of θ is :

A $\frac{4\pi}{9}$

B $\frac{7\pi}{18}$

C $\frac{\pi}{18}$

D $\frac{5\pi}{18}$

Explanation

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2, R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

or $4\sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

3 JEE Main 2021 (Online) 31st August Evening Shift

MCQ (Single Correct Answer)

If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos \gamma)y + (\cos \beta)z = 0$

$(\cos \gamma)x + y + (\cos \alpha)z = 0$

$(\cos \beta)x + (\cos \alpha)y + z = 0$

has :

A no solution

B infinitely many solution

C exactly two solutions

D a unique solution

Explanation

Given $\alpha + \beta + \gamma = 2\pi$

$$\Delta = \begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$$= 1 - \cos^2\alpha - \cos\gamma(\cos\gamma - \cos\alpha\cos\beta) + \cos\beta(\cos\alpha\cos\gamma - \cos\beta)$$

$$= 1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2\cos\alpha\cos\beta\cos\gamma$$

$$= \sin^2\alpha - \cos^2\beta - \cos\gamma(\cos\gamma - 2\cos\alpha\cos\beta)$$

$$= -\cos(\alpha+\beta)\cos(\alpha-\beta) - \cos\gamma(\cos(2\pi-(\alpha-\beta)) - 2\cos\alpha\cos\beta)$$

$$= -\cos(2\pi-\gamma)\cos(\alpha-\beta) - \cos\gamma(\cos(\alpha+\beta) - 2\cos\alpha\cos\beta)$$

$$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma(\cos\alpha\cos\beta + \sin\alpha\sin\beta)$$

$$= -\cos\gamma\cos(\alpha-\beta) + \cos\gamma\cos(\alpha-\beta)$$

$$= 0$$

So, the system of equation has infinitely many solutions.

1 JEE Main 2021 (Online) 31st August Morning Shift

MCQ (Single Correct Answer)

If the following system of linear equations

2x + y + z = 5

x − y + z = 3

x + y + az = b

has no solution, then :

A $a = -\frac{1}{3}, b \neq \frac{7}{3}$

B $a \neq \frac{1}{3}, b = \frac{7}{3}$

C $a \neq -\frac{1}{3}, b = \frac{7}{3}$

D $a = \frac{1}{3}, b \neq \frac{7}{3}$

Explanation

Here $D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} \begin{matrix} = 2(a-1) - 1(a-1) + 1 + 1 \\ = 1 - 3a \end{matrix}$

$$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} \begin{matrix} = 2(-b-3) - 1(b-3) + 5(1+1) \\ = 7 - 3b \end{matrix}$$

for $a = \frac{1}{3}, b \neq \frac{7}{3}$, system has no solutions.

2 JEE Main 2021 (Online) 27th August Evening Shift

MCQ (Single Correct Answer)

Let $[\lambda]$ be the greatest integer less than or equal to λ. The set of all values of λ for which the system of linear equations

x + y + z = 4,

3x + 2y + 5z = 3,

9x + 4y + (28 + $[\lambda]$)z = $[\lambda]$ has a solution is :

A R

B $(-\infty, -9) \cup (-9, \infty)$

C $[-9, -8)$

D $(-\infty, -9) \cup [-8, \infty)$

Explanation

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 9 & 4 & 28+[\lambda] \end{vmatrix} = -24 - [\lambda] + 15 = -[\lambda] - 9$$

if $[\lambda] + 9 \neq 0$ then unique solution

if $[\lambda] + 9 = 0$ then $D_1 = D_2 = D_3 = 0$

so infinite solutions

Hence, λ can be any red number.

2 JEE Main 2021 (Online) 1st September Evening Shift

MCQ (Single Correct Answer)

Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

A $n(S_1) = 2, n(S_2) = 2$

B $n(S_1) = 1, n(S_2) = 0$

C $n(S_1) = 2, n(S_2) = 0$

D $n(S_1) = 0, n(S_2) = 2$

Explanation

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$$

$= -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a-3)(a-4)]$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$$

$= a + 35 - 4 + 14a$

$= 15a + 31$

Now, $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0 \therefore a = 3, a = 4$ and for $a = 3$ and 4, $\Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore n(S_2) = 0$