Practice Questions

Q1.

Page-79

19. Using matrix method, solve the system of equations

$$3x + 2y - 2z = 3,\ x + 2y + 3z = 6,\ 2x - y + z = 2.$$

Sol. Given system of equations is:

$$3x + 2y - 2z = 3$$
$$x + 2y + 3z = 6$$

and $\quad 2x - y + z = 2$

or $\quad AX = B$

i.e. $$\begin{bmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 2 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix}$$

$\therefore \quad X = A^{-1}B$

For A^{-1},

Cofactors are

Cofactors are

$$A_{11} = 5, A_{12} = 5, A_{13} = -5,$$
$$A_{21} = 0, A_{22} = 7, A_{23} = 7,$$
$$A_{31} = 10, A_{32} = -11 \text{ and } A_{33} = 4$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 5 & 5 & -5 \\ 0 & 7 & 7 \\ 10 & -11 & 4 \end{bmatrix}^T = \begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

$$|A| = 3(5) + 2(5) + (-2)(-5) = 35$$

$$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 5 & 0 & 10 \\ 5 & 7 & -11 \\ -5 & 7 & 4 \end{bmatrix}\begin{bmatrix} 3 \\ 6 \\ 2 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 15+20 \\ 15+42-22 \\ -15+42+8 \end{bmatrix} = \frac{1}{35}\begin{bmatrix} 35 \\ 35 \\ 35 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore \quad x = 1, y = 1 \text{ and } z = 1$

Q2.

Page-79

18. If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the system of linear equations $x - 2y = 10$, $2x - y - z = 8$, $-2y + z = 7$.

Sol. We have, $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$ (i)

Cofactors are:

$A_{11} = -3, A_{12} = 2, A_{13} = 2,$
$A_{21} = -2, A_{22} = 1, A_{23} = 1,$
$A_{31} = -4, A_{32} = 2, A_{33} = 3$

$$\therefore \quad \text{adj}A = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix}^T = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

$$|A| = 1(-3) - 2(-2) + 0 = 1$$

$$\therefore \quad A^{-1} = \frac{\text{adj}\,A}{|A|} = \begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$$

Now the system of linear equations is

$x - 2y = 10,$
$2x - y - z = 8$
and $-2y + z = 7$
or $AX = B$

$$\text{i.e.,} \quad \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\text{where,} \quad A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B + \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix}$$

$$\therefore \quad X = A^{-1} B$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 2 \\ -2 & 1 & 1 \\ -4 & 2 & 3 \end{bmatrix} \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} = \begin{bmatrix} -30+16+14 \\ -20+8+7 \\ -40+16+21 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \\ -3 \end{bmatrix}$$

$$\therefore \quad x = 0, y = -5 \text{ and } z = -3$$

Q3.

20. Given $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, then find BA and use this to solve the system of equations $y + 2z = 7$, $x - y = 3$, $2x + 3y + 4z = 17$.

Sol. We have, $A = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$

$$\therefore \quad BA = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$$

$$\therefore \quad B^{-1} = \frac{A}{6} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \qquad \text{(i)}$$

Given system of equations is:

$x - y = 3$, $2x + 3y + 4z = 17$ and $y + 2z = 7$

$$\text{or} \quad \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\therefore \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}^{-1}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 6 + 34 - 28 \\ -12 + 34 - 28 \\ 6 - 17 + 35 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$\therefore \quad x = 2, y = -1$ and $z = 4$