[20]

(3) In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $3\sin x - 4\sin^3 x - k = 0$, $0 < k < 1$, then the measure of angle C is

(JEE-1990)

(A) $\frac{\pi}{3}$ (B) $\frac{5\pi}{6}$ (C) $\frac{2\pi}{3}$ (D) $\frac{3\pi}{4}$

Sol. Given that $A > B$ & $3\sin x - 4\sin^3 x - k = 0$,

$\Rightarrow \sin 3x = k$ $[k \in (0,1)]$

As A & B satisfy above equation

$\therefore \sin 3A = k, \sin 3B = k$

$\Rightarrow \sin 3A - \sin 3B = 0$

$\Rightarrow 2\cos\left(\frac{3A+3B}{2}\right) = 0$ or $\sin\left(\frac{3A-3B}{2}\right) = 0$

$\Rightarrow \frac{3A+3B}{2} = 90^\circ$ or $\frac{3A-3B}{2} = 0$

$\Rightarrow A+B = 60^\circ$ or $A = B$

Given that $A > B \therefore A \neq B$

Thus, $A+B = 60^\circ \Rightarrow \boxed{C = 120^\circ}$

$\Leftrightarrow$ $\boxed{(C)\ \frac{2\pi}{3}}$

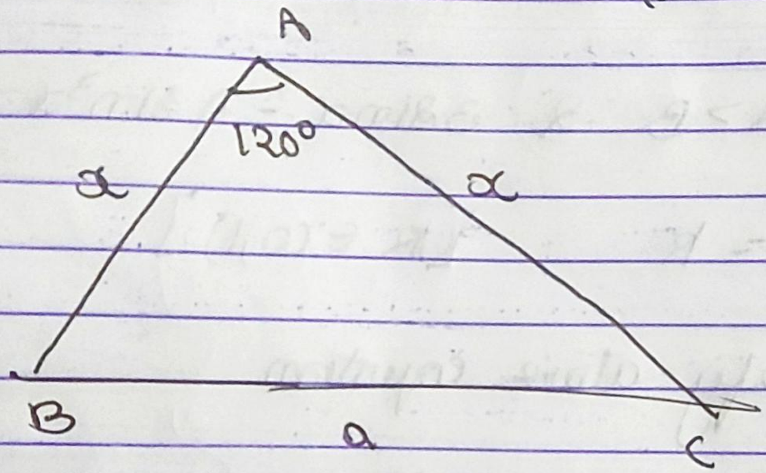
② One angle of an isosceles Δ is 120° and radius of its incircle $= \sqrt{3}$, then the area of the triangle in sq. units is :

(2006 - 3M)

(a) $7+12\sqrt{3}$ (b) $12-7\sqrt{3}$

(c) $12+7\sqrt{3}$ (D) 4π

Sol.



By sine law in ΔABC

$$\frac{x}{\sin 30^\circ} = \frac{a}{\sin 120^\circ} \Rightarrow a = x\sqrt{3}$$

$$\therefore \Delta = \frac{1}{2} \times x \times x \sin 120^\circ = \frac{\sqrt{3}}{4} x^2$$

$$\& \sqrt{3} = \frac{\Delta}{s} \Rightarrow \frac{(2x+a)\sqrt{3}}{2} = \frac{\sqrt{3}}{4} x^2$$

$$x = 2(2+\sqrt{3}) \Rightarrow \Delta = \frac{\sqrt{3}}{4} \times 4(4+3+4\sqrt{3})$$