

14.32 A body of mass m is situated in a potential field $U(x) = U_0 (1 - \cos \alpha x)$ when U_0 and α are constants. Find the time period of small oscillations.

SOLUTION:

GIVEN $U(x) = U_0(1 - \cos \alpha x)$

We know that $F = -\frac{\partial U}{\partial x} = -U_0 \alpha \sin \alpha x$

Now for small 'x', we can write $\sin \alpha x \approx \alpha x$

so, $F = -U_0 \alpha (\alpha x) \Rightarrow -U_0 \alpha^2 x$

which is a equation of SHM

so, $a = -\left(\frac{U_0 \alpha^2}{m}\right) x$ where $\omega^2 = \frac{U_0 \alpha^2}{m}$

i.e $\omega = \alpha \sqrt{\frac{U_0}{m}}$

so, $T = \frac{2\pi}{\omega} = \frac{2\pi}{\alpha} \sqrt{\frac{m}{U_0}}$