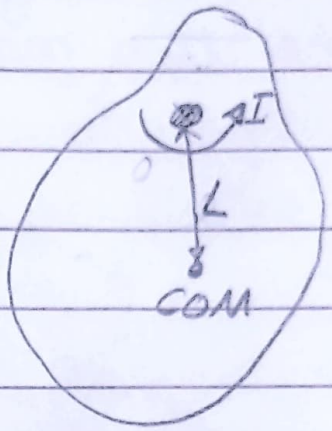
48. Find the time period of small oscillations of the following systems. (a) A metre stick suspended through the 20 cm mark. (b) A ring of mass m and radius r suspended through a point on its periphery. (c) A uniform square plate of edge a suspended through a corner. (d) A uniform disc of mass m and radius r suspended through a point $r/2$ away from the centre.

HC VERMA

SOLUTION : FOR RIGID BODY MOTION:

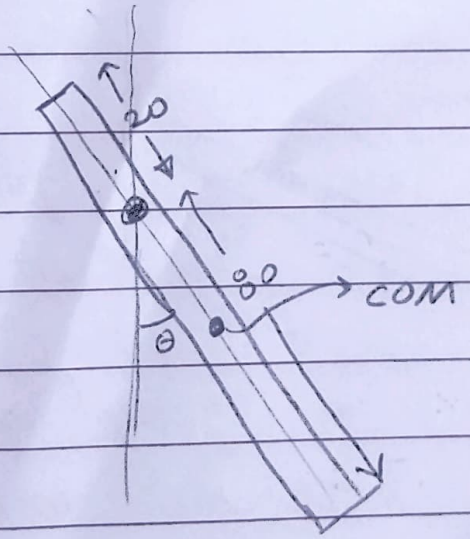
$$T = 2\pi\sqrt{\frac{I}{mgL}}$$



I ≡ moment of inertia about point of suspension.

L ≡ distance of point of suspension from COM.

(a) for metre stick



$l = 50 - 20 = 30\,cm = d$

$I = \frac{ml^2}{12} + md^2$ ($d = 30\,cm$)

I_{COM}

$l = 100\,cm$

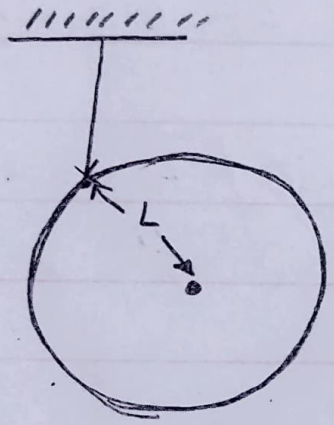
$$T = 2\pi\sqrt{\frac{\frac{ml^2}{12} + md^2}{mgd}}$$

$$T = 2\pi\sqrt{\frac{l^2 + d}{12gd\ g}} = 2\pi\sqrt{\frac{1}{12 \times 10 \times 0.3} + \frac{0.3}{10}}$$

$$= 2\pi\sqrt{0.0577} \Rightarrow 1.509$$

≈ 1.51 sec

(b)



$L = R$

$I = mR^2 + mL^2$

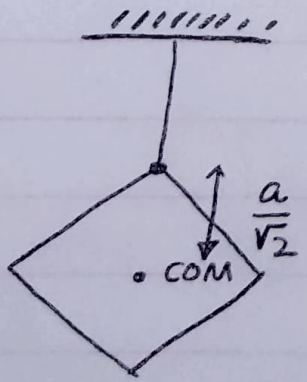
$= mR^2 + mR^2 = 2mR^2$

so,

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{2mR^2}{mgR}}$$

$$= 2\pi\sqrt{\frac{2R}{g}}$$

(c)



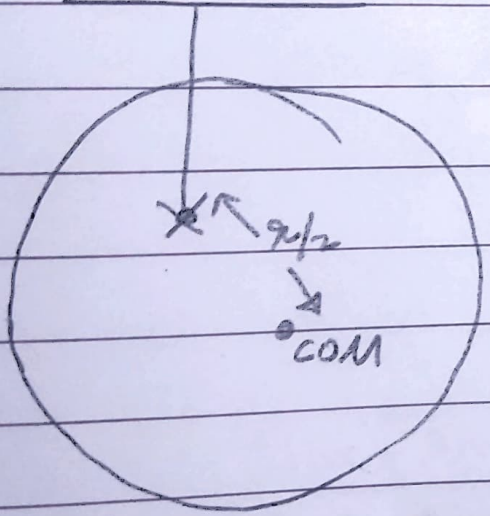
$L = \frac{a}{\sqrt{2}}$ $\quad I = \frac{ma^2}{6} + m\left(\frac{a}{\sqrt{2}}\right)^2$

$\frac{ma^2}{6} \rightarrow I_{COM}$ $\quad m\left(\frac{a}{\sqrt{2}}\right)^2 \rightarrow md^2$

$$I = \frac{2ma^2}{3}$$

$$T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{2ma^2}{mg3a}\sqrt{2}} \Rightarrow 2\pi\sqrt{\frac{2\sqrt{2}a}{3g}}$$

(d)



$L = \frac{r}{2}$ $\quad I = \frac{mr^2}{2} + m\left(\frac{r}{2}\right)^2$

I_{COM} $\quad md^2$

$$I = \frac{mr^2}{2} + \frac{mr^2}{4} = \frac{3mr^2}{4}$$

so, $T = 2\pi\sqrt{\frac{I}{mgL}} = 2\pi\sqrt{\frac{3mr^2 2}{4mgr}}$

$$T = 2\pi\sqrt{\frac{3r}{2g}}$$