31. A uniform plate of mass M stays horizontally and symmetrically on two wheels rotating in opposite directions (figure 12-E16). The separation between the wheels is L. The friction coefficient between each wheel and the plate is μ. Find the time period of oscillation of the plate if it is slightly displaced along its length and released.

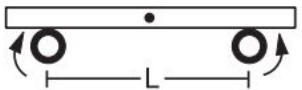
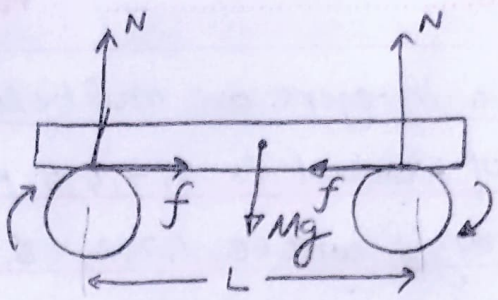


Figure 12-E16

HC VERMA

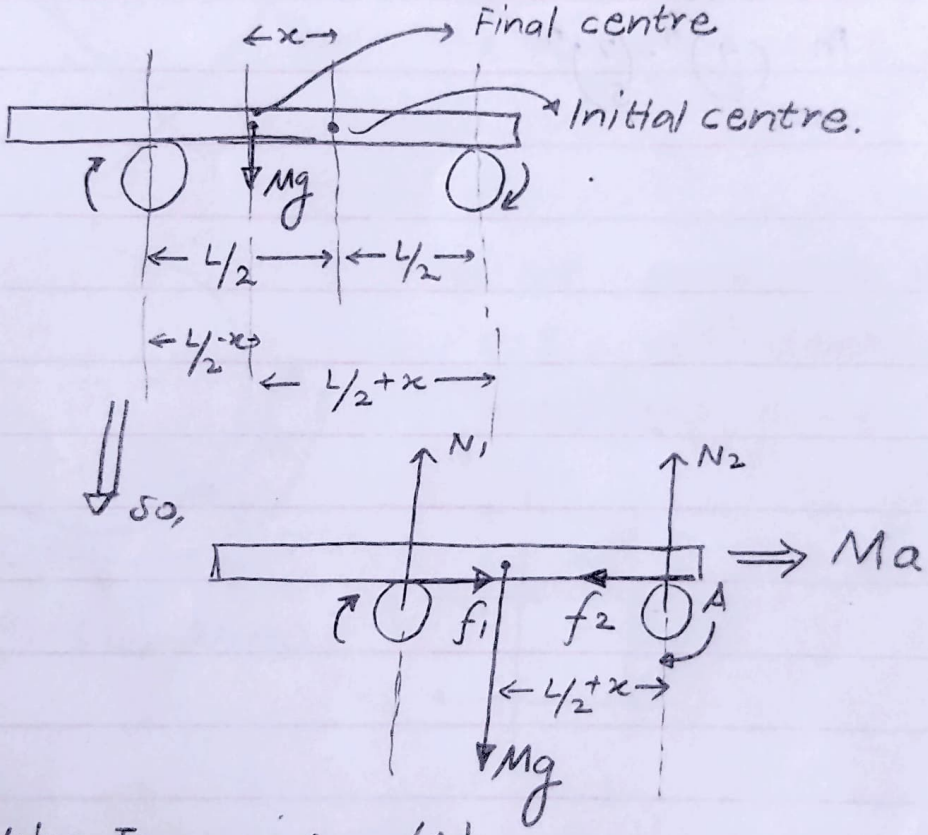
SOLUTION:



Step 1: Analyse Equilibrium

$N = \frac{Mg}{2}$ Not exciting !!

Step-2: Displace by 'x' & then find restoring acceleration.



Taking Torque about 'A'

$$\text{So, } N_1 \times L = Mg\left(\frac{L}{2}+x\right) \Rightarrow N_1 = Mg\left(\frac{1}{2}+\frac{x}{L}\right)$$

$$\text{NOW } N_1 + N_2 = Mg, \text{ so } N_2 = Mg\left(\frac{1}{2}-\frac{x}{L}\right)$$

$$\text{Now, } Ma = f_1 - f_2 \Rightarrow \mu(N_1 - N_2)$$

$$= \mu\left(\frac{Mg}{2} + \frac{Mgx}{L} - \frac{Mg}{2} + \frac{Mgx}{L}\right)$$

$$\text{So, } Ma = \mu \frac{2Mgx}{L} \Rightarrow a = \left(\frac{2\mu g}{L}\right)x$$

so, this acceleration is restoring. so

$$\vec{a} = -\left(\frac{2\mu g}{L}\right)\vec{x}$$

so, $\omega^2 = \frac{2\mu g}{L}$ $\quad T = 2\pi\sqrt{\frac{L}{2\mu g}}$