15. The block of mass m_1 shown in figure (12-E2) is fastened to the spring and the block of mass m_2 is placed against it. (a) Find the compression of the spring in the equilibrium position. (b) The blocks are pushed a further distance $(2/k)\ (m_1 + m_2)g\sin\theta$ against the spring and released. Find the position where the two blocks separate. (c) What is the common speed of blocks at the time of separation ?

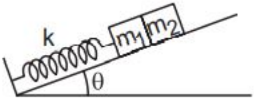
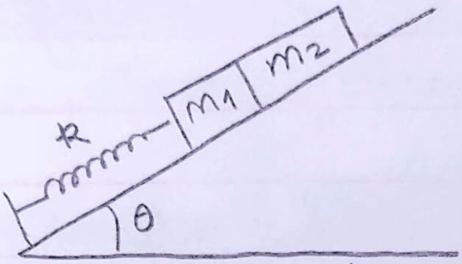


Figure 12-E2

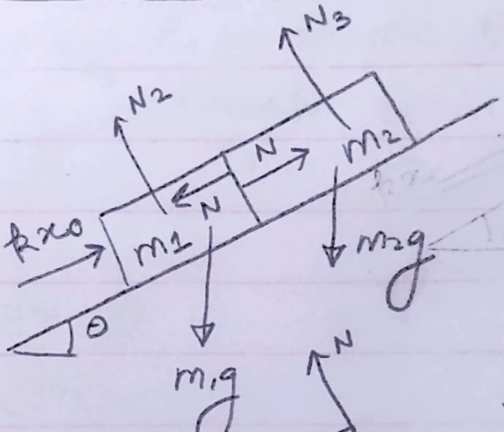
HC VERMA

SOLUTION:

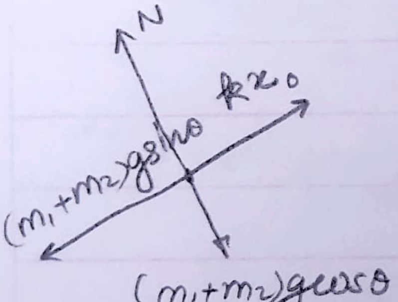
(a) finding compression at equilibrium.



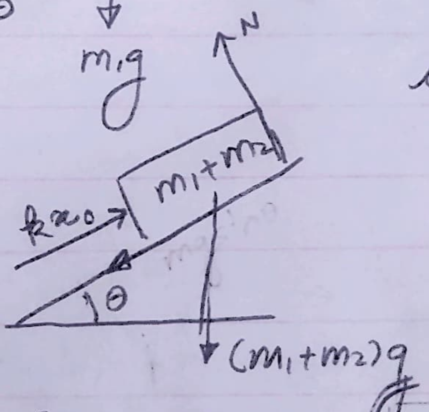
Let compression be 'x_0' FBD of masses.



BOTH masses experience same 'N', so we can combine both masses into one



⇐



so,

$(m_1+m_2)g\sin\theta = kx_0 \Rightarrow$ $\boxed{x_0 = \frac{(m_1+m_2)g\sin\theta}{k}}$

(b) Since the blocks are pushed

$\frac{2(m_1+m_2)g\sin\theta}{k}$ ← This will be the extreme position,

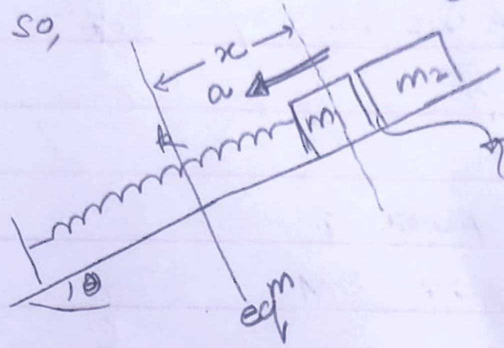
i.e. $A = \frac{2(m_1+m_2)g\sin\theta}{k}$

AMPLITUDE

Now, we can see that if the particle crosses the equilibrium point moving upwards, then only the two masses will separate.

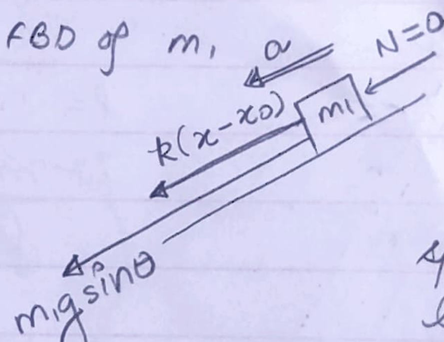
At separation, contact force = 0,

so,



Let's suppose, separation occurs at 'x'.

so, FBD of m_1



for spring force at eq^m, the spring was compressed by 'x_0' which is then extended by 'x'

Total extension $= (x - x_0)$

so, $m_1 a = m_1 g\sin\theta + k(x - x_0)$

now at 'x' $a = +\omega^2 x \Rightarrow \dfrac{kx}{(m_1+m_2)}$

so,

$$\frac{m_1 kx}{m_1+m_2} = m_1 g\sin\theta + kx - kx_0$$

{ Before the separat^n both blocks were involved in SHM, so $\omega = \sqrt{\dfrac{k}{m_1+m_2}}$

so, $$\left(\frac{m_1}{m_1+m_2} - 1\right)kx = m_1 g\sin\theta - \frac{\cancel{k}(m_1+m_2)g\sin\theta}{\cancel{k}}$$

$$\Rightarrow \frac{\cancel{-}m_2 kx}{m_1+m_2} = \cancel{-}m_2 g\sin\theta \Rightarrow \boxed{x = \frac{(m_1+m_2)g\sin\theta}{k}}$$

Now since this $x = x_0$

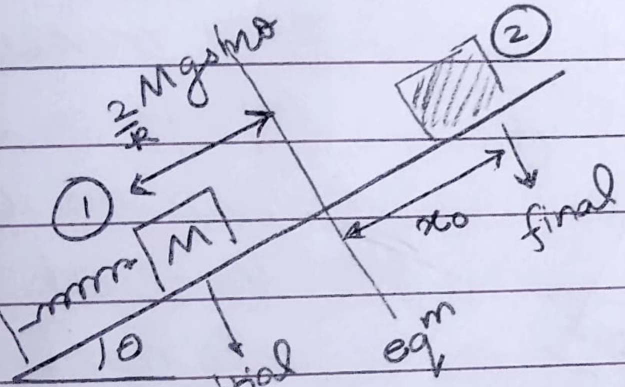
we have extension finally

$x - x_0 = '0'$

So, the spring is at natural length when both masses separate.

(c) Common speed at time of separation.

{ Taking both blocks as a system with mass $M = m_1 + m_2$ }



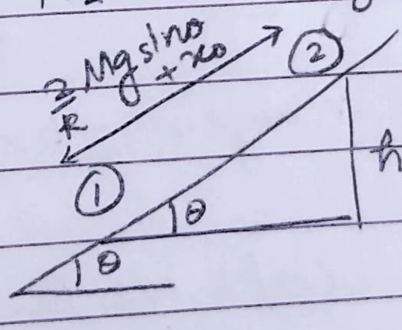
so, $KE_1 = 0$

$KE_2 = \frac{1}{2} M v^2$

$PE_1 = +\frac{1}{2} k \left(\frac{2}{k} Mg\sin\theta + x_0\right)^2$

natural length

$PE_2 = 0 + Mgh$



$\sin\theta = \dfrac{h}{\frac{2}{k} Mg\sin\theta + x_0}$

so, now

$KE_1 + PE_1 = KE_2 + PE_2$

$0 + \frac{2}{k} M^2 g^2 \sin^2\theta + \frac{k x_0^2}{2} + \frac{2k}{k} Mg\sin\theta\, x_0 = \frac{1}{2} M v^2 + Mgh$

$\cancel{\frac{2}{k} M^2 g^2 \sin^2\theta} + \frac{k x_0^2}{2} + 2Mg\sin\theta\, x_0 = \frac{1}{2} M v^2 + \cancel{\frac{2M^2 g^2 \sin^2\theta}{k}} + Mg x_0 \sin\theta$

$$\frac{kx_0^2}{2} + Mg\sin\theta\, x_0 = \frac{1}{2}Mv^2$$

$$\frac{kM^2g^2\sin^2\theta}{2k^2} + \frac{M^2g^2\sin^2\theta}{k} = \frac{1}{2}Mv^2$$

$$\frac{3}{\cancel{2}}\frac{M^{\cancel{2}}g^2\sin^2\theta}{k} = \frac{1}{\cancel{2}}\cancel{M}v^2$$

$$v = \sqrt{\frac{3M}{k}}\, g\sin\theta$$

$$\boxed{v = \sqrt{\frac{3(m_1+m_2)}{k}}\, g\sin\theta}$$