7. Consider a particle moving in simple harmonic motion according to the equation

$$x = 2{\cdot}0 \cos(50 \pi t + \tan^{-1} 0{\cdot}75)$$

where x is in centimetre and t in second. The motion is started at $t = 0$. (a) When does the particle come to rest for the first time ? (b) When does the acceleration have its maximum magnitude for the first time ? (c) When does the particle come to rest for the second time ?

SOLUTION:

(a) finding when particle comes to rest.

i.e velocity $=0$.

$$\frac{dx}{dt} = v = -2 \times 50\pi \sin(50\pi t + \tan^{-1}0.75) = 0$$

$$\Rightarrow \quad \sin[50\pi t + \tan^{-1}(0.75)] = 0$$

so, $\sin x = 0$ when $x = \pm n\pi$

Since 't' cannot be negative, so taking +ve values.

$$50\pi t + \tan^{-1}(0.75) = n\pi$$

We need to find minimum 't', so taking $n=1$ { for $n=0$, t becomes $-ve$ }

$$50\pi t + \tan^{-1}(0.75) = \pi$$

$$t = \frac{\pi - \tan^{-1}(0.75)}{50\pi} = 0.0158$$

$$\approx 1.6 \times 10^{-2} \text{ sec.}$$

(b) acceleration max for first time

$$a = \frac{dv}{dt} = -2 \times 50\pi \times 50\pi \cos(50\pi t + \tan^{-1}0.75) = 0$$

so, for a_{max}, we need $\cos x = \pm 1$

and for minimum value $x = 0$ { here $t < 0$ }

so, $x = \pi$

i.e $50\pi t + \tan^{-1}(0.75) = \pi$

$$t = \frac{\pi - \tan^{-1}(0.75)}{50\pi} = 1.6 \times 10^{-2} \text{ sec}$$

So, when particle comes to rest { $v=0$ } acceleration is more.

(c) for second time rest,
from (a), we have
$$50\pi t + \tan^{-1}(0.75) = n\pi$$
here n=2. {of first time n=1}
so,
$$t = \frac{2\pi - \tan^{-1}(0.75)}{50\pi} = 0.0358 \approx 3.6 \times 10^{-2} \text{ sec.}$$

Second way :

After first rest, we know that particle will rest again after $\frac{T}{2}$

so, $T = \frac{2\pi}{\omega}$ where $\omega = 50\pi$ (from equation)

$$= \frac{2\pi}{50\pi} \Rightarrow 4 \times 10^{-2} \text{ sec.}$$

so, $\frac{T}{2} = 2 \times 10^{-2}$ sec.

so, final answer $= T_{\text{first time rest}} + \frac{T}{2}$

$$= 1.6 \times 10^{-2} + 2 \times 10^{-2} = 3.6 \times 10^{-2} \text{ sec.}$$

first rest   eqm   second rest