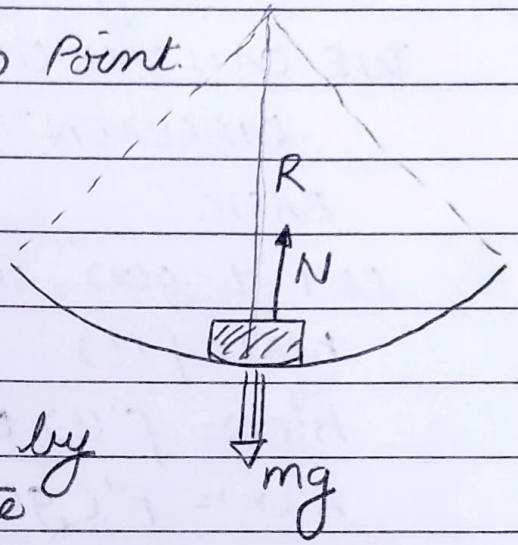
38. A small block oscillates back and forth on a smooth concave surface of radius R (figure 12-E17). Find the time period of small oscillation.

Figure 12-E17

HC VERMA

SOLUTION: We need to calculate the time period
Let's follow the steps:

Step 1: Analyse Equilibrium Point

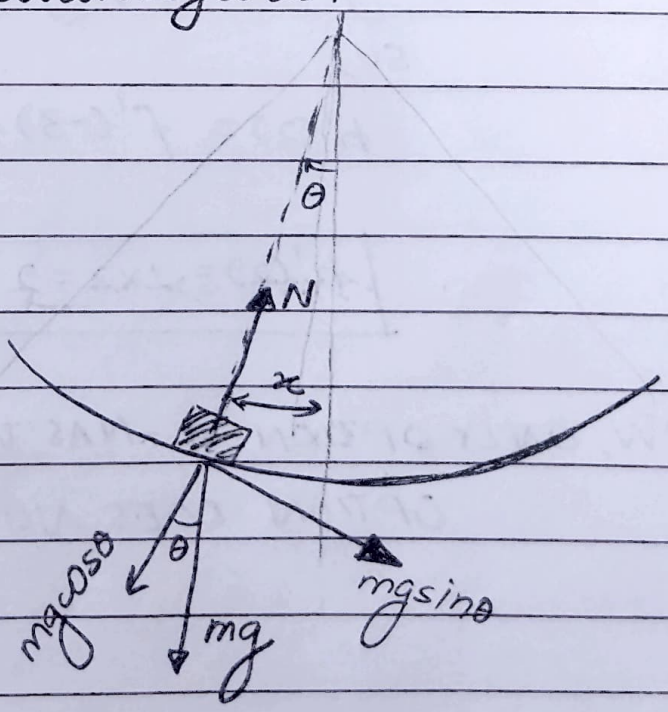


So, $N = mg$

Nothing exciting!!

Step 2: Displace the mass by 'x' and then calculate restoring force to find the acceleration (restoring)

* In this case displacing by angular displacement 'θ' seems more advantageous.



So, the restoring force comes from $mg\sin\theta$
{as N cancels $mg\cos\theta$}

So,

$-ma = mg\sin\theta$

and from figure for short arc

$\sin\theta \approx \theta = \frac{x}{R}$

So, $-ma = \frac{mgx}{R} \Rightarrow a = -\frac{g}{R}x$ which follows SHM

so, from general equation,

$$a = -\omega^2 x$$

we get $\omega^2 = \sqrt{\frac{g}{R}}$ & $T = \frac{2\pi}{\omega} \Rightarrow 2\pi\sqrt{\frac{R}{g}}$

ANSWER#