initial velocity increases the stopping distance by a factor of 4 (for the same deceleration).

For the car of a particular make, the braking distance was found to be 10 m, 20 m, 34 m and 50 m corresponding to velocities of 11, 15, 20 and 25 m/s which are nearly consistent with the above formula.

Stopping distance is an important factor considered in setting speed limits, for example, in school zones. ◀

▶ ***Example 3.8*** **Reaction time :** When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act. For example, if a person is driving and suddenly a boy appears on the road, then the time elapsed before he slams the brakes of the car is the reaction time. Reaction time depends on complexity of the situation and on an individual.

You can measure your reaction time by a simple experiment. Take a ruler and ask your friend to drop it vertically through the gap between your thumb and forefinger (Fig. 3.15). After you catch it, find the distance d travelled by the ruler. In a particular case, d was found to be 21.0 cm. Estimate reaction time.

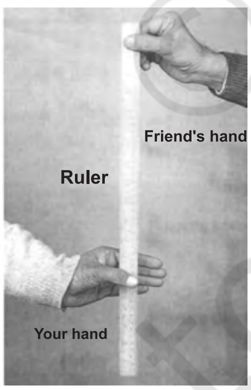

Fig. 3.15 *Measuring the reaction time.*

Answer The ruler drops under free fall. Therefore, $v_o = 0$, and $a = -g = -9.8$ m s^{-2}. The distance travelled d and the reaction time t_r are related by

$$d = -\frac{1}{2} g t_r^2$$

Or, $$t_r = \sqrt{\frac{2d}{g}} \text{ s}$$

Given $d = 21.0$ cm and $g = 9.8$ m s^{-2} the reaction time is

$$t_r = \sqrt{\frac{2 \times 0.21}{9.8}} \text{ s} \cong 0.2 \text{ s.}$$ ◀

3.7 RELATIVE VELOCITY

You must be familiar with the experience of travelling in a train and being overtaken by another train moving in the same direction as you are. While that train must be travelling faster than you to be able to pass you, it does seem slower to you than it would be to someone standing on the ground and watching both the trains. In case both the trains have the same velocity with respect to the ground, then to you the other train would seem to be not moving at all. To understand such observations, we now introduce the concept of relative velocity.

Consider two objects A and B moving uniformly with average velocities v_A and v_B in one dimension, say along x-axis. (Unless otherwise specified, the velocities mentioned in this chapter are measured with reference to the ground). If x_A (0) and x_B (0) are positions of objects A and B, respectively at time $t = 0$, their positions x_A (t) and x_B (t) at time t are given by:

$$x_A(t) = x_A(0) + v_A t \tag{3.12a}$$
$$x_B(t) = x_B(0) + v_B t \tag{3.12b}$$

Then, the displacement from object A to object B is given by

$$x_{BA}(t) = x_B(t) - x_A(t)$$
$$= [\, x_B(0) - x_A(0)\,] + (v_B - v_A)\, t. \tag{3.13}$$

Equation (3.13) is easily interpreted. It tells us that as seen from object A, object B has a velocity $v_B - v_A$ because the displacement from A to B changes steadily by the amount $v_B - v_A$ in each unit of time. We say that the velocity of object B *relative to object* A is $v_B - v_A$:

$$v_{BA} = v_B - v_A \tag{3.14a}$$

Similarly, velocity of object A *relative to object* B is:

$$v_{AB} = v_A - v_B \tag{3.14b}$$

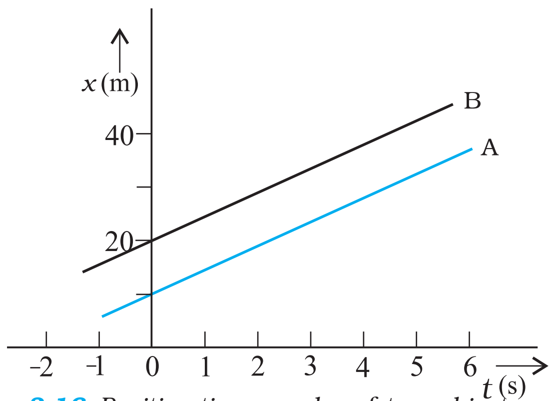

Fig. 3.16 *Position-time graphs of two objects with equal velocities.*

This shows: $\qquad v_{BA} = -\, v_{AB} \qquad$ (3.14c)

Now we consider some special cases :

(a) If $v_B = v_A$, $v_B - v_A = 0$. Then, from Eq. (3.13), $x_B(t) - x_A(t) = x_B(0) - x_A(0)$. Therefore, the two objects stay at a constant distance ($x_B(0) - x_A(0)$) apart, and their position–time graphs are straight lines parallel to each other as shown in Fig. 3.16. The relative velocity v_{AB} or v_{BA} is zero in this case.

(b) If $v_A > v_B$, $v_B - v_A$ is negative. One graph is steeper than the other and they meet at a common point. For example, suppose $v_A = 20$ m s^{-1} and $x_A(0) = 10$ m; and $v_B = 10$ m s^{-1}, $x_B(0) = 40$ m; then the time at which they meet is $t = 3$ s (Fig. 3.17). At this instant they are both at a position $x_A(t) = x_B(t) = 70$ m. Thus, object *A* overtakes object *B* at this time. In this case, v_{BA} = 10 m s^{-1} – 20 m s^{-1} = – 10 m s^{-1} = $-\, v_{AB}$.

(c) Suppose v_A and v_B are of opposite signs. For example, if in the above example object *A* is moving with 20 m s^{-1} starting at $x_A(0) = 10$ m and object *B* is moving with – 10 m s^{-1} starting at $x_B(0) = 40$ m, the two objects meet at $t = 1$ s (Fig. 3.18). The velocity of *B* relative to *A*, v_{BA} = [–10 – (20)] m s^{-1} = –30 m s^{-1} = $-\, v_{AB}$. In this case, the magnitude of v_{BA} or v_{AB} (= 30 m s^{-1}) is greater than the magnitude of velocity of *A* or that of *B*. If the objects under consideration are two trains, then for a person sitting on either of the two, the other train seems to go very fast.

Note that Eq. (3.14) are valid even if v_A and v_B represent instantaneous velocities.

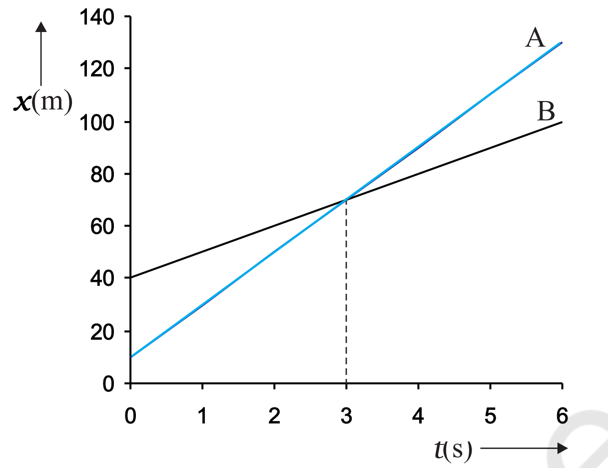

Fig. 3.17 *Position-time graphs of two objects with unequal velocities, showing the time of meeting.*

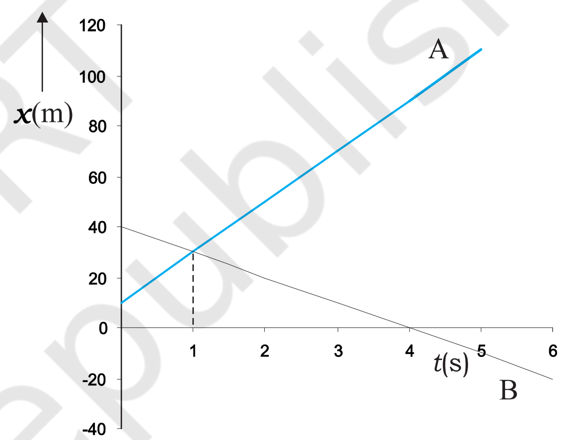

Fig. 3.18 *Position-time graphs of two objects with velocities in opposite directions, showing the time of meeting.*

▶ ***Example 3.9*** *Two parallel rail tracks run north-south. Train A moves north with a speed of 54 km h^{-1}, and train B moves south with a speed of 90 km h^{-1}. What is the*
(a) velocity of B with respect to A ?,
(b) velocity of ground with respect to B ?, and
(c) velocity of a monkey running on the roof of the train A against its motion (with a velocity of 18 km h^{-1} with respect to the train A) as observed by a man standing on the ground ?

Answer Choose the positive direction of *x*-axis to be from south to north. Then,

v_A = + 54 km h^{-1} = 15 m s^{-1}
v_B = – 90 km h^{-1} = – 25 m s^{-1}

Relative velocity of B with respect to $A = v_B - v_A = -40$ m s^{-1}, i.e. the train B appears to A to move with a speed of 40 m s^{-1} from north to south.

Relative velocity of ground with respect to $B = 0 - v_B = 25$ m s^{-1}.

In (c), let the velocity of the monkey with respect to ground be v_M. Relative velocity of the monkey with respect to A,

$v_{MA} = v_M - v_A = -18$ km h^{-1} = –5 m s^{-1}. Therefore, $v_M = (15 - 5)$ m s^{-1} = 10 m s^{-1}.

SUMMARY

1. An object is said to be in *motion* if its position changes with time. The position of the object can be specified with reference to a conveniently chosen origin. For motion in a straight line, position to the right of the origin is taken as positive and to the left as negative.
2. *Path length* is defined as the total length of the path traversed by an object.
3. *Displacement* is the change in position : $\Delta x = x_2 - x_1$. Path length is greater or equal to the magnitude of the displacement between the same points.
4. An object is said to be in *uniform motion* in a straight line if its displacement is equal in equal intervals of time. Otherwise, the motion is said to be *non-uniform*.
5. *Average velocity* is the displacement divided by the time interval in which the displacement occurs :

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

   On an *x-t* graph, the average velocity over a time interval is the slope of the line connecting the initial and final positions corresponding to that interval.
6. *Average Speed* is the ratio of total path length traversed and the corresponding time interval.

   The average speed of an object is greater or equal to the magnitude of the average velocity over a given time interval.
7. *Instantaneous velocity* or simply velocity is defined as the limit of the average velocity as the time interval Δt becomes infinitesimally small :

$$v = \lim_{\Delta t \to 0} \bar{v} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

   The velocity at a particular instant is equal to the slope of the tangent drawn on position-time graph at that instant.
8. *Average acceleration* is the change in velocity divided by the time interval during which the change occurs :

$$\bar{a} = \frac{\Delta v}{\Delta t}$$

9. *Instantaneous acceleration* is defined as the limit of the average acceleration as the time interval Δt goes to zero :

$$a = \lim_{\Delta t \to 0} \bar{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

   The acceleration of an object at a particular time is the slope of the velocity-time graph at that instant of time. For uniform motion, acceleration is zero and the *x-t* graph is a straight line inclined to the time axis and the *v-t* graph is a straight line parallel to the time axis. For motion with uniform acceleration, x-t graph is a parabola while the v-t graph is a straight line inclined to the time axis.

10. The area under the velocity-time curve between times t_1 and t_2 is equal to the displacement of the object during that interval of time.
11. For objects in uniformly accelerated rectilinear motion, the five quantities, displacement x, time taken t, initial velocity v_0, final velocity v and acceleration a are related by a set of simple equations called *kinematic equations of motion :*

$$v = v_0 + at$$

$$x = v_0 t + \frac{1}{2} a t^2$$

$$v^2 = v_0^2 + 2ax$$

if the position of the object at time $t = 0$ is 0. If the particle starts at $x = x_0$, x in above equations is replaced by $(x - x_0)$.

Physical quantity	Symbol	Dimensions	Unit	Remarks
Path length		[L]	m	
Displacement	Δx	[L]	m	$= x_2 - x_1$ In one dimension, its sign indicates the direction.
Velocity		$[LT^{-1}]$	$m\ s^{-1}$	
(a) Average	$\bar{v}$			$= \dfrac{\Delta x}{\Delta t}$
(b) Instantaneous	v			$= \lim\limits_{\Delta t \to 0} \dfrac{\Delta x}{\Delta t} = \dfrac{dx}{dt}$ In one dimension, its sign indicates the direction.
Speed		$[LT^{-1}]$	$m\ s^{-1}$	
(a) Average				$= \dfrac{\text{Path length}}{\text{Time interval}}$
(b) Instantaneous				$= \dfrac{dx}{dt}$
Acceleration		$[LT^{-2}]$	$m\ s^{-2}$	
(a) Average	$\bar{a}$			$= \dfrac{\Delta v}{\Delta t}$
(b) Instantaneous	a			$= \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$ In one dimension, its sign indicates the direction.

POINTS TO PONDER

1. The path length traversed by an object between two points is, in general, not the same as the magnitude of displacement. The displacement depends only on the end points; the path length (as the name implies) depends on the actual path. In one dimension, the two quantities are equal only if the object does not change its direction during the course of motion. In all other cases, the path length is greater than the magnitude of displacement.
2. In view of point 1 above, the average speed of an object is greater than or equal to the magnitude of the average velocity over a given time interval. The two are equal only if the path length is equal to the magnitude of displacement.
3. The origin and the positive direction of an axis are a matter of choice. You should first specify this choice before you assign signs to quantities like displacement, velocity and acceleration.
4. If a particle is speeding up, acceleration is in the direction of velocity; if its speed is decreasing, acceleration is in the direction opposite to that of the velocity. This statement is independent of the choice of the origin and the axis.
5. The sign of acceleration does not tell us whether the particle's speed is increasing or decreasing. The sign of acceleration (as mentioned in point 3) depends on the choice of the positive direction of the axis. For example, if the vertically upward direction is chosen to be the positive direction of the axis, the acceleration due to gravity is negative. If a particle is falling under gravity, this acceleration, though negative, results in increase in speed. For a particle thrown upward, the same negative acceleration (of gravity) results in decrease in speed.
6. The zero velocity of a particle at any instant does not necessarily imply zero acceleration at that instant. A particle may be momentarily at rest and yet have non-zero acceleration. For example, a particle thrown up has zero velocity at its uppermost point but the acceleration at that instant continues to be the acceleration due to gravity.
7. In the kinematic equations of motion [Eq. (3.11)], the various quantities are algebraic, i.e. they may be positive or negative. The equations are applicable in all situations (for one dimensional motion with constant acceleration) provided the values of different quantities are substituted in the equations with proper signs.
8. The definitions of instantaneous velocity and acceleration (Eqs. (3.3) and (3.5)) are exact and are always correct while the kinematic equations (Eq. (3.11)) are true only for motion in which the magnitude and the direction of acceleration are constant during the course of motion.

EXERCISES

3.1 In which of the following examples of motion, can the body be considered approximately a point object:
(a) a railway carriage moving without jerks between two stations.
(b) a monkey sitting on top of a man cycling smoothly on a circular track.
(c) a spinning cricket ball that turns sharply on hitting the ground.
(d) a tumbling beaker that has slipped off the edge of a table.

3.2 The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in Fig. 3.19. Choose the correct entries in the brackets below ;
(a) (A/B) lives closer to the school than (B/A)
(b) (A/B) starts from the school earlier than (B/A)
(c) (A/B) walks faster than (B/A)
(d) A and B reach home at the (same/different) time
(e) (A/B) overtakes (B/A) on the road (once/twice).