$$\lim_{x \to +\infty} \frac{2x^2+3}{x^2-5x-1} = \lim_{x \to +\infty} \frac{x^2\left(2+\frac{3}{x^2}\right)}{x^2\left(1-\frac{5}{x}-\frac{1}{x^2}\right)}$$

$$= \lim_{x \to +\infty} \frac{2+\frac{3}{x^2}}{1-\frac{5}{x}-\frac{1}{x^2}}$$

$$= \frac{2+0}{1-0-0}$$

$$\lim_{x \to +\infty} \frac{2x^2+3}{x^2-5x-1} = 2$$

3. *Check the continuity of the following function at 2.*

$$f(x)=\begin{cases}\frac{1}{2}(x^2-4) \ if\ 0<x<2\\ 0,\ if\ x=2 \ and\ 2-8x^{-3},\ if\ x>2\end{cases}$$

Solution:

$$f(x)=\begin{cases}\frac{1}{2}(x^2-4)\ if\ 0<x<2\\ 0,\ if\ x=2\ and\ 2-8x^{-3},\ if\ x>2\end{cases}$$

$If\ a=2$

$$LHL=\lim_{x\to a^-} f(x)=\lim_{x\to 2^-} f(x)$$

$$=\lim_{x\to a^-}\frac{1}{2}(x^2-4)=\frac{1}{2}(2^2-4)$$

$$=\frac{1}{2}(4-4)=\frac{1}{2}\times 0=0$$

$$RHL=\lim_{x\to a^+} f(x)=\lim_{x\to 2^+} f(x)$$

$$=\lim_{x\to 2^+} 2-8x^{-3}=2-8(2)^{-3}$$

$$=2-8\times\frac{1}{8}=2-1=1$$

$\therefore LHL=RHL$

$\therefore \lim_{x\to 2} f(x)$ *does not rxists.*

$\therefore f$ *is discontinuous at 2*

12. *Compute* $\lim_{x \to a} \frac{x \sin a - a \sin x}{x - a}$.

Solution :

$$\lim_{x \to a} \frac{x \sin a - a \sin x}{x - a}$$

Add and subtract, $a \sin a$ *in the numerator*

$$= \lim_{x \to a} \frac{x \sin a + a \sin a - a \sin a - a \sin x}{x - a}$$

$$= \lim_{x \to a} \frac{(x - a) \sin a - a (\sin a - \sin x)}{x - a}$$

$$= sin\, a \lim_{x \to a} \frac{(x - a)}{x - a} + a \lim_{x \to a} \frac{\sin a - \sin x}{x - a}$$

$$= \sin a(1) + a \lim_{x \to a} \frac{2 \cos\left(\frac{a + x}{2}\right) \sin\left(\frac{a - x}{2}\right)}{x - a}$$

$$= \sin a + 2a \lim_{x \to a} \cos\left(\frac{a + x}{2}\right) \lim_{x \to a} \frac{\sin\left(\frac{a - x}{2}\right)}{-(a - x)}$$

$$= \sin a + 2a \cos\left(\frac{a + a}{2}\right) \lim_{x \to a} \frac{\sin\left(\frac{a - x}{2}\right)}{\frac{a - x}{2} \times 2}$$

$$= \sin a + 2a \cos\left(\frac{2a}{2}\right) . \frac{1}{2} \lim_{\frac{a - x}{2} \to a} \frac{\sin\left(\frac{a - x}{2}\right)}{\frac{a - x}{2}}$$

$$\left(\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right)$$

$$= \sin a - a \cos a\,(1) = \sin a - a \cos a$$

5. $\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$

Solution : $\lim_{x \to 5} \frac{\sqrt{x+4}-3}{x-5}$

Multiplying and dividing by $\sqrt{x+4}+3$ we get,

$$= \lim_{x \to 5} \frac{(x+4)-3^2}{(x-5)\left[\sqrt{x+4}+3\right]} = \lim_{x \to 5} \frac{(x+4)-9}{(x-5)\left[\sqrt{x+4}+3\right]}$$

$$= \lim_{x \to 5} \frac{\cancel{x-5}}{\cancel{(x-5)}\left[\sqrt{x+4}+3\right]} = \lim_{x \to 5} \frac{1}{\sqrt{x+4}+3}$$

14. *Compute* $\lim_{x\to 2} \frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)}$.

Solution :

$$\lim_{x\to 2} \frac{2x^2-7x-4}{(2x-1)(\sqrt{x}-2)}$$

$$\lim_{x\to 2} \frac{(x-4)(2x+1)}{(2x-1)(\sqrt{x}-2)}$$

$$\lim_{x\to 2} \frac{(x-4)(2x+1)}{(2x-1)(\sqrt{x}-2)} \times \frac{(\sqrt{x}+2)}{(\sqrt{x}+2)}$$

$$\lim_{x\to 2} \frac{(x-4)(2x+1)(\sqrt{x}+2)}{(2x-1)(x-4)}$$

$$\lim_{x\to 2} \frac{(2x+1)(\sqrt{x}+2)}{(2x-1)}$$

$$= \frac{(2(2)+1)(\sqrt{2}+2)}{2(2)-1} = \frac{5(\sqrt{2}+2)}{3}$$