28. **If** $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$ **then find the value of k.**

Sol. Given that $\lim_{x \to 1} \frac{x^4 - 1}{x - 1} = \lim_{x \to k} \frac{x^3 - k^3}{x^2 - k^2}$

$$\Rightarrow 4(1)^{4-1} = \lim_{x \to k} \frac{(x-k)(x^2 + k^2 + kx)}{(x-k)(x+k)}$$

$$\Rightarrow 4 = \lim_{x \to k} \frac{x^2 + k^2 + kx}{x + k} \Rightarrow 4 = \frac{k^2 + k^2 + k^2}{2k}$$

$$\Rightarrow 4 = \frac{3k^2}{2k} \Rightarrow 4 = \frac{3}{2}k \Rightarrow k = \frac{8}{3}$$

Hence, the required value of k is $\frac{8}{3}$.

25. Evaluate: $\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

Sol. Given that $\lim_{x \to \frac{\pi}{6}} \frac{\cot^2 x - 3}{\operatorname{cosec} x - 2}$

$$= \lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 1 - 3}{\operatorname{cosec} x - 2} = \lim_{x \to \frac{\pi}{6}} \frac{\operatorname{cosec}^2 x - 4}{\operatorname{cosec} x - 2}$$

$$= \lim_{x \to \frac{\pi}{6}} \frac{(\operatorname{cosec} x + 2)(\operatorname{cosec} x - 2)}{(\operatorname{cosec} x - 2)} = \lim_{x \to \frac{\pi}{6}} (\operatorname{cosec} x + 2)$$

Taking limit we have

$$= \operatorname{cosec} \frac{\pi}{6} + 2 = 2 + 2 = 4$$

Hence, the required answer is 4.

15. Evaluate: $\lim_{x \to 0} \frac{\sin 3x}{\sin 7x}$

Sol. Given that $\lim_{x \to 0} \frac{\sin 3x}{\sin 7x}$

$$= \lim_{x \to 0} \frac{\frac{\sin 3x}{3x} \times 3x}{\frac{\sin 7x}{7x} \times 7x} = \frac{\lim_{3x \to 0}\left(\frac{\sin 3x}{3x}\right)}{\lim_{7x \to 0}\left(\frac{\sin 7x}{7x}\right)} \times \frac{3}{7}$$

$$= \frac{1}{1} \times \frac{3}{7} = \frac{3}{7} \left[\because \lim_{x \to 0} \frac{\sin x}{x} = 1\right]$$

Hence, the required answer is $\frac{3}{7}$.

60. $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$ is

(a) 2

(b) 0

(c) 1

(d) -1

Sol. Given $\lim_{x\to 0} \frac{\sin x}{\sqrt{x+1}-\sqrt{1-x}}$

$$= \lim_{x\to 0} \frac{\sin x\left[\sqrt{x+1}+\sqrt{1-x}\right]}{(\sqrt{x+1}-\sqrt{1-x})(\sqrt{x+1}+\sqrt{1-x})}$$

$$= \lim_{x\to 0} \frac{\sin x\left[\sqrt{x+1}+\sqrt{1-x}\right]}{x+1-1+x}$$

$$= \lim_{x\to 0} \frac{\sin x\left[\sqrt{x+1}+\sqrt{1-x}\right]}{2x} = \frac{1}{2}.\lim_{x\to 0} \frac{\sin x}{x}[\sqrt{x+1}+\sqrt{1-x}]$$

Taking limit, we get

$$= \frac{1}{2}\times 1\times\left[\sqrt{0+1}+\sqrt{0-1}\right] = \frac{1}{2}\times 1\times 2 = 1$$

Hence, the correct option is **(c)**.

61. $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$ is equal to

(a) 3

(b) 1

(c) 0

(d) 2

Sol. Given, $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{1 + \tan^2 x - 2}{\tan x - 1}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\tan^2 x - 1}{\tan x - 1} = \lim_{x \to \frac{\pi}{4}} \frac{(\tan x + 1)(\tan x - 1)}{(\tan x - 1)}$

$= \lim_{x \to \frac{\pi}{4}} (\tan x + 1) = \tan \frac{\pi}{4} + 1 = 1 + 1 = 2$

Hence, the correct option is **(d)**.

63. If f(x) = $\begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$ **where [.] denotes the greatest integer function. Then** $\lim_{x \to 0}$ **f(x)**

(a) 1

(b) 0

(c) -1

(d) None of these

Sol. Given, f(x) = $\begin{cases} \frac{\sin[x]}{[x]}, & [x] \neq 0 \\ 0, & [x] = 0 \end{cases}$

LHL = $\lim_{x \to 0^-} \frac{\sin[x]}{[x]} = \lim_{h \to 0} \frac{\sin[0-h]}{[0-h]} = \lim_{h \to 0} \frac{-\sin[-h]}{[-h]} = -1$