Q. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then –

(A) a = 1, b = 4 (B) a = 1, b = –4] (C) a = 2, b = –3 (D) a = 2, b = 3

[JEE 2012, 3M, –1M]

Sol. (B)

$$\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{(1-a)x^2+x(1-a-b)+1-b}{x+1}\right)=4$$

$$\Rightarrow \lim_{x\to\infty}\frac{\left((1-a).x+1-a-b+\left(\frac{1-b}{x}\right)\right)}{1+\frac{1}{x}}=4$$

for limit to exist finitely

$1-a=0$ and $1-a-b=4$

$\Rightarrow a=1$ and $b=-4$.

Q. Let $f(x) = \frac{1-x(1+|1-x|)}{|1-x|} \cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$.

Then

(A) $\lim_{x \to 1^+} f(x)$ does not exist

(B) $\lim_{x \to 1^-} f(x)$ does not exist

(C) $\lim_{x \to 1^-} f(x) = 0$

(D) $\lim_{x \to 1^+} f(x) = 0$

[JEE(Advanced)-2017]

Sol. (A,C)

$$f(x)=\begin{cases}(1-x)\cos\dfrac{1}{1-x}, & x<1\\ -(1+x)\cos\dfrac{1}{1-x}, & x>1\end{cases}$$

$$\lim_{x\to 1^+} f(x)=\text{d.n.e}, \lim_{x\to 1^-} f(x)=0$$