

$$\int \frac{(3x+2)dx}{(x+1)(x+3)}$$

$$\frac{3x+2}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}$$

$$3x+2 = A(x+3) + B(x+1)$$

$$3x+2 = (A+B)x + (3A+B)$$

$$A+B=3, \quad 3A+B=2$$

On solving we get

$$A = -\frac{1}{2}, \quad B = \frac{7}{2}$$

$$\int \frac{(3x+2)dx}{(x+1)(x+3)} = \int \frac{-dx}{2(x+1)} + \int \frac{7dx}{2(x+3)}$$

$$= \frac{7}{2} \ln|x+3| - \frac{\ln|x+1|}{2} + C$$