Q) $\int \frac{1-\cos x}{\cos x(1+\cos x)}\,dx$

$$I = \int \frac{1-\cos x}{\cos x(1+\cos x)}\,dx = \int \frac{2\sin^2 x/2}{2\cos^2 x/2 \cdot (2\cos^2 x/2 - 1)}\,dx$$

$$I = \int \frac{\tan^2 x/2 \cdot \sec^2 x/2}{\sec^2 x/2 (2\cos^2 x/2 - 1)}\,dx$$

$$I = \int \frac{\tan^2 x/2 \sec^2 x/2}{(2 - \sec^2 x/2)}\,dx$$

$$\tan x/2 = t$$

$$\frac{1}{2}\sec^2 x/2\,dx = dt$$

$$I = \int \frac{2t^2\,dt}{(1-t^2)} = \int \frac{2dt}{1-t^2} - \int 2\,dt$$

$$= 2\ln\left|\frac{1+t}{1-t}\right| - 2t + C$$

$$= 2\ln\left|\frac{1+\tan x/2}{1-\tan x/2}\right| - 2\tan x/2 + C$$

On simplifying we get :-

$$2\ln|\sec x + \tan x| - 2\tan x/2 + C$$