Q) $\displaystyle\int \frac{\sin x}{\sin 4x}\,dx$

$$I = \int \frac{\sin x\,dx}{\sin 4x} = \int \frac{\cancel{\sin x}\,dx}{4\cancel{\sin x}\cos x\cos 2x}$$

$$= \int \frac{\cos x\,dx}{4\cos^2 x\,(1-2\sin^2 x)} = \int \frac{\cos x\,dx}{4(1-\sin^2 x)(1-2\sin^2 x)}$$

let $\sin x = t$

$\cos x\,dx = dt$

then

$$I = \int \frac{dt}{4(1-t^2)(1-2t^2)}$$

$$\frac{1}{(1-t^2)(1-2t^2)} = \frac{A}{1-t^2} + \frac{B}{1-2t^2}$$

$$1 = A(1-2t^2) + B(1-t^2)$$

$$1 = (A+B) - (2A+B)t^2$$

Here, $A+B=1,\ 2A+B=0$

On solving we get $A=-1,\ B=2$

So, $$I = \int \frac{2\,dt}{4(1-2t^2)} - \int \frac{dt}{4(1-t^2)}$$

$$I = \frac{1}{4\sqrt{2}} \log\left|\frac{1+\sqrt{2}t}{1-\sqrt{2}t}\right| - \frac{1}{8}\log\left|\frac{1+t}{1-t}\right| + c$$

$$I = \frac{1}{4\sqrt{2}} \log\left|\frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| - \frac{1}{8}\log\left|\frac{1+\sin x}{1-\sin x}\right| + c$$