Q) Four persons independently solve a certain problem with probabilities $1/2, 3/4, 1/4, 1/8$. Then the probability that the problem is solved atleast one of them is

JEE 2013, 2M

(A) $\frac{235}{256}$ (B) $\frac{21}{256}$ (C) $\frac{21}{256}$ (D) $\frac{253}{256}$

- P(problem solved by atleast one of them)

$= 1 - P(\text{solved by none})$

$= 1 - \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4} \times \frac{7}{8}\right) = 1 - \frac{21}{256} = \frac{235}{256}$

✓(A)

Q) Of the three independent events E_1, E_2 and E_3, the probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let the probability p that none of the events E1, E2 or E3 occurs satisfy the equations $(\alpha - 2\beta)P = \alpha\beta$ and $(\beta - 3\gamma)P = 2\beta\gamma$. All the given probabilities are assumed to lie in the interval $(0,1)$. Then $\frac{\text{Probability of occurrence of E1}}{\text{Probability of occurrence of E3}} =$

JEE 2013, 4(-1)

- $P(E_1) = P_1$, $P(E_2) = P_2$, $P(E_3) = P_3$

given that, $P_1(1-P_2)(1-P_3) = \alpha$ —— ①

$P_2(1-P_1)(1-P_3) = \beta$ —— ②

$P_3(1-P_1)(1-P_2) = \gamma$ —— ③

and $(1-P_1)(1-P_2)(1-P_3) = P$ —— (4)

$$\Rightarrow \frac{P_1}{1-P_1} = \frac{\alpha}{P}, \quad \frac{P_2}{1-P_2} = \frac{\beta}{P} \quad \& \quad \frac{P_3}{1-P_3} = \frac{\gamma}{P}$$

Also, $\beta = \frac{\alpha P}{\alpha + 2P} = \frac{3\gamma P}{P - 2\gamma}$

$$\Rightarrow \alpha P - 2\alpha\gamma = 3\alpha\gamma + 6P\gamma$$

$$\Rightarrow \frac{P_1}{1-P_1} - \frac{6P_3}{1-P_3} = \frac{5P_1P_3}{(1-P_1)(1-P_3)}$$

$$\Rightarrow P_1 - 6P_3 = 0$$

$$\frac{P_1}{P_3} = 6$$

(Q) A box B_1 contains 1 white ball, 3 red balls and 2 black balls. Another box B_2 contains 2 white balls, ~~and~~ 3 red balls and 4 black balls. A third box B_3 contains 3 white balls, 4 red balls and 5 black balls.

JEE 2013 3(-1)

(a) If 2 balls are drawn (without replacement) from a randomly selected box and one of the balls is white and other ball is red, the probability that these 2 balls are drawn from box B_2 is

(A) $\frac{116}{181}$ (B) $\frac{126}{181}$ (C) $\frac{65}{181}$ (D) $\frac{55}{181}$

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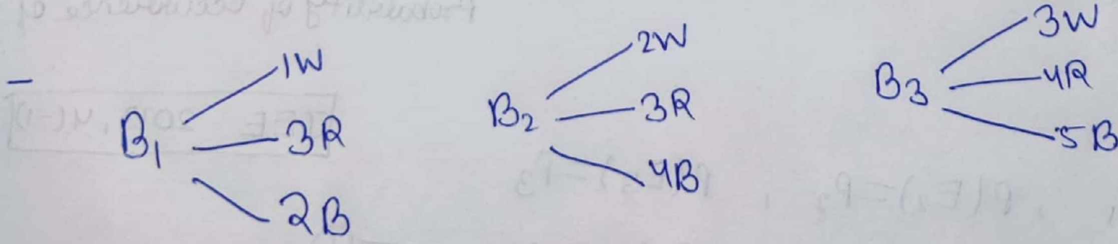

$$P(B_1) = P(B_2) = P(B_3) = \frac{1}{3}$$

$$P(A|B_1) = \frac{{}^1C_1 \times {}^3C_1}{{}^6C_2}, \quad P(A|B_2) = \frac{{}^2C_1 \times {}^3C_1}{{}^9C_2}$$

$$P(A|B_3) = \frac{{}^3C_1 \times {}^4C_1}{{}^9C_2}$$

$$P(B_2|A) = \frac{P(A|B_2)\,P(B_2)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2) + P(A|B_3)P(B_3)}$$

$$= \frac{55}{181}$$

(D) ✓

(b) If 1 ball is drawn from each of the boxes B_1, B_2 and B_3, the probability that all 3 drawn balls are of the same colour is

(A) $\frac{82}{648}$ (B) $\frac{90}{648}$ (C) $\frac{558}{648}$ (D) $\frac{566}{648}$

- Probability of 3 drawn balls of same colour

$$= \frac{1}{6} \times \frac{2}{9} \times \frac{3}{12} + \frac{3}{6} \times \frac{3}{9} \times \frac{4}{12} + \frac{2}{6} \times \frac{4}{9} \times \frac{5}{12}$$

$$= \frac{82}{648}$$

(Q) Three boys and two girls stand in a queue. The probability that the number of boys ahead of every girl is at least one more than the number of girls ahead of her, is

JEE 2014, 3(-1)

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

- Total ways of arranging $= 5! = 120$

(I) _ _ _ _ _ g g } $2 \cdot 4! = 48$

_ _ _ _ g g

(II) _ _ _ g g B } $2! \cdot 3! = 12$

Favourable way are $120 - 48 - 12 = 60$

$$P = \frac{60}{120} = \frac{1}{2}$$

(Q) Box 1 contains three cards bearing numbers, 1,2,3; Box 2 contains five cards bearing numbers 1,2,3,4,5; and Box 3 contains seven cards bearing numbers 1,2,3,4,5,6,7. A card is drawn from each of the boxes. let x_i be the number on the card drawn from the ith box, i=1,2,3

JEE 2014, 3(-1)

(a) The probability that $x_1 + x_2 + x_3$ is odd, is

(A) $\frac{29}{105}$ (B) $\frac{53}{106}$ (C) $\frac{57}{105}$ (D) $\frac{1}{2}$

- $x_1 + x_2 + x_3 \rightarrow$ Odd

Odd + odd + odd $\Rightarrow \frac{2}{3} \cdot \frac{3}{5} \cdot \frac{4}{7} = \frac{24}{105}$

odd + even + even $\Rightarrow \frac{2}{3} \cdot \frac{2}{5} \cdot \frac{3}{7} = \frac{12}{105}$

even + odd + even $\Rightarrow \frac{1}{3} \cdot \frac{3}{5} \cdot \frac{3}{7} = \frac{9}{105}$

even + even + odd $\Rightarrow \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{4}{7} = \frac{8}{105}$

total $= \frac{24+12+9+8}{105} = \frac{53}{105}$

(b) The probability that x_1, x_2, x_3 are in arithmetic progression is

(A) $\frac{9}{105}$ (B) $\frac{10}{105}$ (C) $\frac{11}{105}$ (D) $\frac{7}{105}$

- $2x_2 = x_1 + x_3$

$x_1 + x_3 \rightarrow$ even for every x_2

even + even $\Rightarrow \left(\frac{1}{3} \cdot \frac{3}{7}\right) \frac{1}{5} = \frac{3}{105}$

odd + odd $\Rightarrow \left(\frac{2}{3}\cdot\frac{4}{7}\right)\frac{1}{5} = \frac{8}{105}$

total $= \frac{8+3}{105} = \frac{11}{105}$

Ⓠ The minimum no. of times a fair coin needs to be tossed, so that the probability of getting at least two heads is at least 0.96 is

JEE 2015, 4(-0)

- Let no. of tosses be n

Probability of getting atleast two heads

$= 1 - \left(\frac{1}{2}\right)^n - {}^nC_1\left(\frac{1}{2}\right)^{n-1}\cdot\frac{1}{2}$

$\Rightarrow 1 - \frac{n+1}{2^n} \geq \frac{24}{25}$

$\Rightarrow \frac{n+1}{2^n} \leq \frac{1}{25}$ $\quad$ $n=8$