# Complex Number and Quadratic Equation

**Complex number:-** A number in the form of $x+iy$, where $x$ and $y$ are real number and $i=\sqrt{-1}$ is called complex number.

C, the set of complex number = $\{x+iy : x,y \in R \text{ and } i=\sqrt{-1}\}$

The complex number is generally denoted by $z$.

Thus $z = x+iy$.

**Conjugate of a complex number**

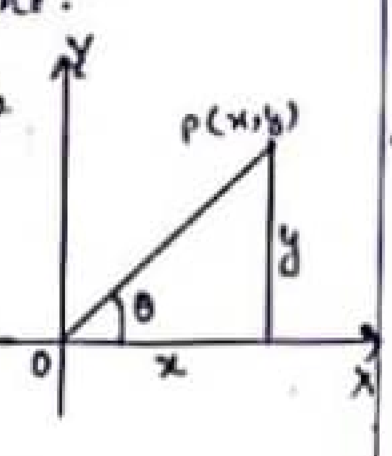
When two complex number differ only in the sign of $i$, they are said to be conjugate of each other.

Thus $x+iy$ and $x-iy$ are two conjugate complex number.

The conjugate of a complex number $z$ is denoted by $\bar{z}$.

**Representation of complex number in Argand plane:** If $z=x+iy$ is non-zero complex number. which is represented by $P(x,y)$ in the Argand plane.

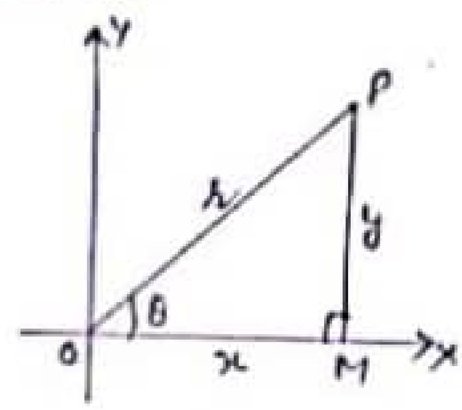
**Modulus** of $z = +\sqrt{x^2+y^2}$ which is real number and represent by $|z|$




**Amplitude** (or Argument) of $z$ is the angle, which OP make with the positive direction of x-axis and is denoted by amp(z) or (arg z).

**Polar Representation:-**

Let $z = x+iy$




Here $\cos\theta = \frac{OM}{OP}$

$OM = OP\cos\theta$

$x = r\cos\theta$.

and $\sin\theta = \frac{MP}{OP}$

$MP = OP\sin\theta$

$y = r\sin\theta$.

where $r = |OP|$

$\therefore z = r\cos\theta + i\,r\sin\theta$

$z = r(\cos\theta + i\sin\theta)$

where $r = \sqrt{x^2+y^2}$

and $\theta = \tan^{-1}\frac{y}{x}$.

## Exercise 5.1

Express each of the following in the form of $a+ib$.

(1) $(5i)(-\frac{3}{5}i)$

Sol: $(5\times -\frac{3}{5})\times(i\times i) = -3\times i^2 = -3\times -1$
$= 3$.

$= a+ib$ where $a=3$ and $b=0$

(2) $i^9+i^{19}$

Sol. $i\,i^8 + i\,i^{18}$

$= i(i^2)^4 + i(i^2)^9 = i(-1)^4 + i(-1)^9$

$= i - i = 0$

$= a+ib$ where $a=0$, $b=0$.

Q3. $i^{-39} = \frac{1}{i^{39}} = \frac{i}{i^{39}\times i} = \frac{i}{(i^2)^{20}}$

$= \frac{i}{(-1)^{20}} = \frac{i}{1} = i = a+ib$ where

$a = 0,\ b = 1$

Q4. $3(7+i7) + i(7+i7)$

Sol $21+21i+7i+7i^2$

$\Rightarrow 21+28i+7(-1) = 21-7+28i$

$= 14+28i$

$= a+ib$ where $a=14,\ b=28$ #

Q5. $(1-i)-(-1+i6)$

Sol :- $1-i+1-i6$

$= 2-7i = a+ib$ where

$a=2,\ b=-7$

Q6. $\left(\frac{1}{5}+i\frac{2}{5}\right) - \left(4+i\frac{5}{2}\right)$

Sol $= \frac{1}{5}+i\frac{2}{5}-4-i\frac{5}{2}$

$= \left(\frac{1}{5}-4\right) + \left(\frac{2}{5}-\frac{5}{2}\right)i$

$= \left(\frac{1-20}{5}\right) + \left(\frac{4-25}{10}\right)i = \frac{-19}{5} - \frac{21}{10}i$

$= a+ib$ where $a = -\frac{19}{5}$ and $b = \frac{-21}{10}$

Q7. $\left[\left(\frac{1}{3}+i\frac{7}{3}\right)\right] + \left[4+i\frac{1}{3}\right] - \left(-\frac{4}{3}+i\right)$

Sol $\frac{1}{3}+i\frac{7}{3}+4+i\frac{1}{3}+\frac{4}{3}-i$

$= \left(\frac{1}{3}+4+\frac{4}{3}\right) + \left(\frac{7}{3}+\frac{1}{3}-1\right)i$

$= \frac{17}{3}+\frac{5}{3}i = a+ib$

where $a=\frac{17}{3},\ b=\frac{5}{3}$.

Q8. $(1-i)^4$

Sol $[(1-i)^2]^2 = (1^2+i^2-2i)^2$

$= (1-1-2i)^2 = (-2i)^2 = 4(-1)$

$= -4 = a+ib$

where $a=-4,\ b=0$

Q9. $\left(\frac{1}{3}+3i\right)^3$ $\left[(a+b)^3 = a^3+b^3+3ab(a+b)\right]$

Sol :- $\left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} + 27i^3 + 3i\left(\frac{1}{3}+3i\right)$

$= \frac{1}{27} + 27i^2 i + i + 9i^2$

$= \frac{1}{27} + 27(-1)i + i + 9(-1)$

$= \left(\frac{1}{27}-9\right) + (i-27i)$

$= \left(\frac{1-243}{27}\right) - 26i = \frac{242}{27} - 26i$

$= a+ib$ where $a = \frac{242}{27}$, $b=-26$

Q10. $\left(-2-\frac{1}{3}i\right)^3$

Sol: $(-1)^3\left(2+\frac{1}{3}i\right)^3$

$= -1\left[(2)^3 + \left(\frac{1}{3}i\right)^3 + 3(2)\left(\frac{1}{3}i\right)\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 + \frac{1}{27}i^2 i + 2i\left(2+\frac{1}{3}i\right)\right]$

$= -\left[8 + \frac{1}{27}(-1)i + 4i + \frac{2}{3}i^2\right]$

$= -\left[8 - \frac{1}{27}i + 4i + \frac{2}{3}(-1)\right]$

$= -\left[8 - \frac{2}{3} + 4i - \frac{1}{27}i\right]$

$= -\left(\frac{22}{3} + \frac{107}{27}i\right) = -\frac{22}{3} - \frac{107}{27}i$ #

Find the multiplicative inverse of each of the complex number:

Q11 $4-3i$

Sol. Multiplicative inverse of $4-3i$ is $\frac{1}{4-3i}$

$= \frac{1}{4-3i} \times \frac{4+3i}{4+3i} = \frac{4+3i}{(4)^2-(3i)^2}$

$= \frac{4+3i}{16-9(-1)} = \frac{4+3i}{16+9} = \frac{4+3i}{25}$

$= \frac{4}{25} + \frac{3i}{25}$.

Q12. $\sqrt{5}+3i$

Sol. Multiplicative inverse of $\sqrt{5}+3i$ is $\frac{1}{\sqrt{5}+3i}$

$= \frac{1}{\sqrt{5}+3i} \times \frac{\sqrt{5}-3i}{\sqrt{5}-3i} = \frac{\sqrt{5}-3i}{(\sqrt{5})^2-(3i)^2}$

$= \frac{\sqrt{5}-3i}{5-9(-1)} = \frac{\sqrt{5}-3i}{5+9} = \frac{\sqrt{5}-3i}{14}$

$= \frac{\sqrt{5}}{14} - \frac{3i}{14}$.

Q13: $-i$

Sol: Multiplicative inverse of $-i$ is $\frac{1}{-i}$

$= \frac{1}{-i} \times \frac{i}{i} = \frac{i}{-i^2} = \frac{i}{-(-1)} = \frac{i}{1} = i$

Q14. Express the following expression in the form of $a+ib$:

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

Sol: $\frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+i\sqrt{2}}$

$= \frac{9-i^2(5)}{2\sqrt{2}i} = \frac{9-(-1)5}{2\sqrt{2}i}$

$= \frac{9+5}{2\sqrt{2}i} = \frac{14}{2\sqrt{2}i} = \frac{7}{\sqrt{2}i}$

$= \frac{7}{\sqrt{2}i} \times \frac{\sqrt{2}i}{\sqrt{2}i} = \frac{7\sqrt{2}i}{(\sqrt{2})^2 i^2}$

$= \frac{7\sqrt{2}i}{2(-1)} = \frac{7\sqrt{2}}{-2}i = a+ib$

where $a=0$, $b=\frac{7\sqrt{2}}{-2}$.

Properties of Amplitudes (or Argument)

(i) If $x>0, y>0$, $z$ lies in first quadrant. and
$\text{amp}(z) = \tan^{-1}\left(\frac{y}{x}\right)$

(ii) When $z$ lies in IInd quad.
$x<0, y>0$.
$\text{amp}(z) = \pi - \tan^{-1}\frac{y}{|x|}$

(iii) When $z$ lies in IIIrd quad.
$x<0, y<0$
$\text{amp}(z) = -\pi + \tan^{-1}\left(\frac{y}{x}\right)$

(iv) When $z$ lies in IV quad.
$x>0, y<0$, $\text{amp}(z) = -\tan^{-1}\left(\frac{|y|}{x}\right)$.

## Exercise 5.2

Find the modulus and argument of each of the following complex number.

1. $z = -1 - i\sqrt{3}$.

Sol: We have $z = -1 - i\sqrt{3}$

let $-1 - i\sqrt{3} = r(\cos\theta + i\sin\theta)$

$-1 = r\cos\theta$ — (1)

$-\sqrt{3} = r\sin\theta$ — (2)

Squaring and adding (1) and (II)

$(-1)^2 + (-\sqrt{3})^2 = r^2\cos^2\theta + r^2\sin^2\theta$

$1 + 3 = r^2(\cos^2\theta + \sin^2\theta)$

$\Rightarrow 4 = r^2 \Rightarrow r = \sqrt{4} = 2$

Dividing (2) by (1)

$\frac{r\sin\theta}{r\cos\theta} = \frac{-\sqrt{3}}{-1}$

$\tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$

$\theta = \frac{\pi}{3}$

Also $\cos\theta = -\frac{1}{2}$, $\sin\theta = -\frac{\sqrt{3}}{2}$

Thus $\theta$ lies in III quadrant

$\therefore \theta = -\pi + \frac{\pi}{3} = -\frac{3\pi + \pi}{3} = -\frac{2\pi}{3}$

Hence $|z| = 2$, $\arg(z) = -\frac{2\pi}{3}$.

(2) $-\sqrt{3} + i$

We have: $z = -\sqrt{3} + i$

let $-\sqrt{3} + i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -\sqrt{3}$ — (1)

$r\sin\theta = 1$ — (2)

Sq. and adding (1) and (2) we get

$r^2(\cos^2\theta + \sin^2\theta) = (-\sqrt{3})^2 + (1)^2$

$\Rightarrow r^2 = 3 + 1$

$\Rightarrow r = \sqrt{4} = 2$

Dividing (2) by (1)

$\tan\theta = -\frac{1}{\sqrt{3}}$

Also $\cos\theta = -\frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$

$\left[\because r\cos\theta = -\sqrt{3},\ r = 2 \quad \cos\theta = -\frac{\sqrt{3}}{r} = -\frac{\sqrt{3}}{2}\right]$

Thus $\theta$ lies in IInd quad.

$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$

Hence $|z| = 2$, $\arg(z) = \frac{5\pi}{6}$

Convert each of the following complex number in polar form.

3. $1 - i$

Sol: let $1 - i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = 1$ — (1) $r\sin\theta = -1$ — (2)

Sq. and adding

$r^2(\cos^2\theta + \sin^2\theta) = 1 + 1$

$\Rightarrow r^2 = 2 \Rightarrow r = \sqrt{2}$.

Dividing (2) by (1)

$\tan\theta = -\frac{1}{1} = 1 = \tan\frac{\pi}{4}$

Also $\cos\theta = \frac{1}{\sqrt{2}}$, $\sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore \theta$ lies in IV quadrant

$\Rightarrow \theta = -\frac{\pi}{4}$.

Polar form is

$\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right)\right)$ #

Q4. $-1+i$

Sol: let $-1+i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -1, \quad r\sin\theta = 1$

Sq. and adding

$r^2 = (-1)^2 + (1)^2$

$r^2 = 1+1 = 2 \Rightarrow r = \sqrt{2}$

Dividing we get $\tan\theta = -1$

Also $\cos\theta = -\frac{1}{\sqrt{2}}, \quad \sin\theta = \frac{1}{\sqrt{2}}$

$\therefore$ $\theta$ lies in IInd quad.

$\Rightarrow \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}$.

Hence $z = \sqrt{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

Q5 $-1-i$

Sol: let $-1-i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -1, \quad r\sin\theta = -1$

Sq. and adding

$r^2 = (-1)^2 + (-1)^2 = 1+1$

$r^2 = 2 \Rightarrow r = \sqrt{2}$

Dividing we get

$\tan\theta = 1$.

Also $\cos\theta = \frac{-1}{\sqrt{2}}, \quad \sin\theta = -\frac{1}{\sqrt{2}}$

$\therefore$ $\theta$ lies in III quadrant

$\Rightarrow \theta = -\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$

$\therefore$ Hence $z = \sqrt{2}\left(\cos\left(-\frac{3\pi}{4}\right) + i\sin\left(-\frac{3\pi}{4}\right)\right)$

Q6: $-3$

Sol let $-3 = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = -3, \quad r\sin\theta = 0$

Sq. and add

$r^2 = (-3)^2 = 9$

$r = \sqrt{9} = 3$

Dividing $\tan\theta = 0$

Also $\cos\theta = -1, \quad \sin\theta = 0$

Thus $\theta = \pi$

Hence $z = 3(\cos\pi + i\sin\pi)$

Q7. $\sqrt{3}+i$

Sol Let $\sqrt{3}+i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = \sqrt{3}, \quad r\sin\theta = 1$

Sq. and adding

$r^2 = (\sqrt{3})^2 + (1)^2 = 3+1 = 4$

$r = \sqrt{4} = 2$

Dividing we get

$\tan\theta = \frac{1}{\sqrt{3}}$

Also $\cos\theta = \frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2}$

$\Rightarrow$ $\theta$ lies in Ist quadrant

$\Rightarrow \theta = \frac{\pi}{6}$

Hence $\sqrt{3}+i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Q8. $i$

let $i = r(\cos\theta + i\sin\theta)$

$\Rightarrow r\cos\theta = 0, \quad r\sin\theta = 1$

Sq. and adding

$\Rightarrow r^2 = 0+1 = 1 \Rightarrow r = \sqrt{1} = 1$

Dividing we get $\tan\theta = \frac{1}{0} = \tan\frac{\pi}{2}$

$\cos\theta = 0, \quad \sin\theta = 1$

$\Rightarrow \theta = \frac{\pi}{2}$, $\theta$ lies in Ist quad.

Hence $i = 1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$= \left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$ #

## Exercise: 5.3

Solve each of the following equation

1) $x^2+3=0$

Sol: $x^2=-3 \Rightarrow x=\pm\sqrt{-3}=\pm\sqrt{3}\,i$

2). $2x^2+x+1=0$

Sol: $a=2,\ b=1,\ c=1$

$\therefore D= b^2-4ac = 1^2-4(2)1$

$= 1-8=-7$

$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-1\pm\sqrt{-7}}{2(2)}=\dfrac{-1\pm\sqrt{-1\times 7}}{4}$

$x=\dfrac{-1\pm\sqrt{7}\,i}{4}.$

Q7. $\sqrt{2}x+x+\sqrt{2}=0$

Sol: $a=\sqrt{2},\ b=1,\ c=\sqrt{2}$

$\therefore D= 1^2-4(\sqrt{2})(\sqrt{2}) = 1-4(2)$

$=1-8=-7$

$\therefore x\pm \dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\dfrac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}.$

Q9:- $x^2+x+\dfrac{1}{\sqrt{2}}=0$

Sol: $\dfrac{\sqrt{2}x^2+\sqrt{2}x+1}{\sqrt{2}}=0$

$\sqrt{2}x^2+\sqrt{2}x+1=0$

Here $a=\sqrt{2},\ b=\sqrt{2},\ c=1$

$\therefore D=(\sqrt{2})^2-4(\sqrt{2})1 = 2-4\sqrt{2}$

$\therefore x=\dfrac{-\sqrt{2}\pm\sqrt{(\sqrt{2})^2-4(\sqrt{2})1}}{2(\sqrt{2})}$

$=\dfrac{-\sqrt{2}\pm\sqrt{2-4\sqrt{2}}}{2\sqrt{2}}$

$=\dfrac{-\sqrt{2}\pm\sqrt{2}\sqrt{(1-2\sqrt{2})}}{2\sqrt{2}}$

$=\dfrac{\sqrt{2}(-1\pm\sqrt{1-2\sqrt{2}})}{2\sqrt{2}}=\dfrac{-1\pm\sqrt{1-2\sqrt{2}}}{2}$ ++

Q10. $x^2+\dfrac{x}{\sqrt{2}}+1=0$

Sol: $\sqrt{2}x^2+x+\sqrt{2}=0$

$a=\sqrt{2},\ b=1,\ c=\sqrt{2}$

$\therefore D= 1^2-4(\sqrt{2})(\sqrt{2}) = 1-8=-7.$

$\therefore x=\dfrac{-b\pm\sqrt{D}}{2a}=\dfrac{-1\pm\sqrt{-7}}{2\sqrt{2}}=\dfrac{-1\pm\sqrt{7}\,i}{2\sqrt{2}}.$

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## Miscellaneous Exercise

1 Evaluate : $\left[i^{18}+\left(\frac{1}{i}\right)^{25}\right]^3$

Sol: $\left[(i^2)^9+\left(\frac{i}{i\,i}\right)^{25}\right]^3=\left[(-1)^9+\left(\frac{i}{-1}\right)^{25}\right]^3$

$=\left[-1+(-i)^{25}\right]^3=\left[-1+(i\,i^{24})(-1)^{25}\right]^3$

$=\left[-1-1(i)\right]^3=\left[-1-i\right]^3=-1(1+i)^3$

$\cancel{= -1[1+i^2+2(1)i] = -1[1+i-1+2i]}$

$\cancel{= -1[1+i]}.$

$=-1\left[(1)^3+i^3+3(1)(i)(1+i)\right]$

$=-\left[1-i+3i(1+i)\right]$

$=-1\left(1-i+3i+3(-1)\right]=-1\left[+2i+1-3\right]$

$=\cancel{-1[-2+2i]}\Rightarrow -1\left(+2i-2\right]=2-2i$

Q2: For any two complex number $z_1$ and $z_2$ prove that :

$Re(z_1z_2)=Re\,z_1\,Re\,z_2-Im\,z_1\,Im\,z_2.$

Sol: Let $z_1=a+ib$, $z_2=c+id$

$a=Re\,z_1$, $c=Re\,z_2$

$b=Im\,z_1$, $d=Im\,z_2$

Now $z_1z_2=(a+ib)(c+id)$

$=(ac-bd)+i(ad+bc).$

$\therefore Re(z_1 z_2) = ac - bd$

$= Re z_1 Re z_2 - Im z_1 Im z_2$.

Q3. Reduce $\left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$ to standard form.

Sol: Here $\frac{1}{1-4i} = \frac{1}{1-4i} \times \frac{1+4i}{1+4i}$

$= \frac{1+4i}{1^2 - (4i)^2} = \frac{1+4i}{1-16i^2} = \frac{1+4i}{1-16(-1)}$

$= \frac{1+4i}{1+16} = \frac{1+4i}{17} = \frac{1}{17} + \frac{4}{17} i$.

$\frac{2}{1+i} = \frac{2}{1+i} \times \frac{1-i}{1-i} = \frac{2(1-i)}{(1)^2 - (i)^2}$

$= \frac{2(1-i)}{1-(-1)} = \frac{2(1-i)}{1+1} = \frac{2(1-i)}{2}$

$= 1 - i$.

and $\frac{3-4i}{5+i} = \frac{3-4i}{5+i} \times \frac{5-i}{5-i}$

$= \frac{15 - 3i - 20i + 4i^2}{5^2 - (i)^2} = \frac{15 - 4 - 23i}{25 - (-1)}$

$= \frac{11 - 23i}{25+1} = \frac{11-23i}{26} = \frac{11}{26} - \frac{23}{26} i$

$\therefore \left(\frac{1}{1-4i} - \frac{2}{1+i}\right)\left(\frac{3-4i}{5+i}\right)$

$= \left[\left(\frac{1}{17} + \frac{4}{17} i\right) - (1-i)\right]\left[\left(\frac{11}{26} - \frac{23}{26} i\right)\right]$

$= \left[\left(\frac{1}{17} - 1\right) + \left(\frac{4}{17} + 1\right) i\right]\left[\frac{11}{26} - \frac{23}{26} i\right]$

$= \left[\left(\frac{1-17}{17}\right) + \left(\frac{4+17}{17}\right) i\right]\left[\frac{11}{26} - \frac{23}{26} i\right]$

$= \left(-\frac{16}{17} + \frac{21}{17} i\right)\left(\frac{11}{26} - \frac{23}{26} i\right)$

$= -\frac{16}{17} \times \frac{11}{26} + \frac{16}{17} \times \frac{23}{26} i + \frac{21}{17} \times \frac{11}{26} i - \frac{21}{17} \times \frac{23}{26} i^2$.

$= \frac{-176}{442} + \frac{368}{442} i + \frac{231}{442} i + \frac{483}{442}$

$= \left(-\frac{176}{442} + \frac{483}{442}\right) + \left(\frac{368}{442} + \frac{231}{442}\right) i$

$= \frac{307}{442} + \frac{599}{442} i$ #

Q3: If $x - iy = \sqrt{\frac{a-ib}{c-id}}$, prove that $(x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$.

We have $x - iy = \sqrt{\frac{a-ib}{c-id}}$ — (1)

Changing $i$ into $-i$

$x + iy = \sqrt{\frac{a+ib}{c+id}}$ — (ii)

Multiply (1) and (ii)

$\Rightarrow x^2 - i^2 y^2 = \sqrt{\frac{(a-ib)(a+ib)}{(c-id)(c+id)}}$

$\Rightarrow x^2 + y^2 = \sqrt{\frac{a^2 - i^2 b^2}{c^2 - i^2 d^2}}$

$= x^2 + y^2 = \sqrt{\frac{a^2+b^2}{c^2+d^2}}$. Sq. b/s.

$\therefore (x^2+y^2)^2 = \frac{a^2+b^2}{c^2+d^2}$

Q5. Convert into polar form:

(i) $\dfrac{1+7i}{(2-i)^2}$

Sol: $\dfrac{1+7i}{(2)^2+i^2-2(2)i} = \dfrac{1+7i}{4-1-4i}$

$= \dfrac{1+7i}{3-4i} \times \dfrac{3+4i}{3+4i} = \dfrac{3+4i+21i+28i^2}{(3)^2-(4i)^2}$

$= \dfrac{3+25i-28}{9+16}$ $\quad [\because i^2=-1]$

$= \dfrac{-25+25i}{25} = \dfrac{(-1+i)25}{25} = -1+i$

Let $-1+i = r(\cos\theta + i\sin\theta)$

Here $r\cos\theta = -1$, $r\sin\theta = 1$

Sq. and adding we get

$r^2 = 1+1 = 2$

$r = \sqrt{2}$.

Dividing we get $\tan\theta = -1$

$\cos\theta = -\dfrac{1}{\sqrt{2}}$, $\quad \sin\theta = \dfrac{1}{\sqrt{2}}$

$\Rightarrow \theta = \pi - \dfrac{\pi}{4} = \dfrac{4\pi - \pi}{4} = \dfrac{3\pi}{4}$.

$\therefore$ Polar form is $\sqrt{2}\left(\cos\dfrac{3\pi}{4} + i\sin\dfrac{3\pi}{4}\right)$.

(ii) $\dfrac{1+3i}{1-2i} = \dfrac{1+3i}{1-2i} \times \dfrac{1+2i}{1+2i}$

$= \dfrac{1+2i+3i+6i^2}{1^2-(2i)^2} = \dfrac{1-6+5i}{1+4}$

$= \dfrac{-5+5i}{5} = \dfrac{5(-1+i)}{5} = -1+i$

Now it is same as part (i).

Solve each of the equation in

(6) $3x^2 - 4x + \dfrac{20}{3} = 0$

Sol: $a=3,\ b=-4,\ c=\dfrac{20}{3}$

$9x^2 - 12x + 20 = 0$

$a = 9,\ b = -12,\ c = 20$

$D = b^2 - 4ac = (-12)^2 - 4(9)(20)$

$= 144 - 720 = -576$.

$\therefore x = \dfrac{-b \pm \sqrt{D}}{2a} = \dfrac{12 \pm \sqrt{-576}}{2\times 9}$

$= \dfrac{12 \pm 24i}{18} = \dfrac{6(2 \pm 4i)}{6\times 3} = \dfrac{2 \pm 4i}{3}$.

Hence $x = \dfrac{2}{3} \pm \dfrac{4i}{3}$.

Q10: $z_1 = 2-i$, $z_2 = 1+i$, find $\left|\dfrac{z_1+z_2+1}{z_1-z_2+i}\right|$

Sol: $\dfrac{z_1+z_2+1}{z_1-z_2+i} = \dfrac{(2-i)+(1+i)+1}{2-i-(1+i)+i}$

$= \dfrac{2-i+1+i+1}{2-i-1-i+i} = \dfrac{4}{1-i}$

$= \dfrac{4}{1-i} \times \dfrac{1+i}{1+i} = \dfrac{4(1+i)}{1^2-i^2}$

$= \dfrac{4(1+i)}{1+1} = \dfrac{4}{2}(1+i) = 2+2i$

$\therefore \left|\dfrac{z_1+z_2+1}{z_1-z_2+i}\right| = \sqrt{2^2+2^2} = \sqrt{4+4}$

$= \sqrt{8} = 2\sqrt{2}$.

Q11. $a+ib = \frac{(x+i)^2}{2x^2+1}$, prove that

$$a^2+b^2 = \frac{(x^2+1)^2}{(2x^2+1)^2}.$$

Solution: $a+ib = \frac{(x+i)^2}{2x^2+1}$

$$= \frac{x^2+i^2+2xi}{2x^2+1} = \frac{x^2-1}{2x^2+1} + \frac{2x}{2x^2+1}i$$

By comparing

$$a = \frac{x^2-1}{2x^2+1}, \quad b = \frac{2x}{2x^2+1}$$

Sq. and adding $\therefore a^2+b^2$

$$\frac{(x^2-1)^2}{(2x^2+1)^2} + \frac{(2x)^2}{(2x^2+1)^2}$$

$$= \frac{(x^2)^2+1^2-2x^2+4x^2}{(2x^2+1)^2}$$

$$= \frac{(x^2)^2+1+2x^2}{(2x^2+1)^2} = \frac{(x^2+1)^2}{(2x^2+1)^2}$$

$[\because a^2+b^2+2ab = (a+b)^2]$.

Q12:- If $z_1 = 2-i$, $z_2 = -2+i$.

find: (i) $Re\left(\frac{z_1 z_2}{\bar{z}_1}\right)$

(ii) $Im\left(\frac{1}{z_1\bar{z}_2}\right)$

Solution: Here $z_1 = 2-i$, $z_2 = -2+i$

$\therefore \bar{z}_1 = 2+i$ and $\bar{z}_2 = -2-i$

$$\therefore \frac{z_1 z_2}{\bar{z}_1} = \frac{(2-i)(-2+i)}{2+i} = \frac{-(2-i)^2}{2+i}$$

$$= -\frac{(4+i^2-4i)}{2+i} = -\frac{(4-1-4i)}{2+i}$$

$$= -\frac{(3-4i)}{2+i} = \frac{4i-3}{2+i} \times \frac{2-i}{2-i}$$

$$= \frac{8i-6+4+3i}{(2)^2-i^2} = \frac{-2+11i}{4+1}$$

$$= \frac{-2+11i}{5} = -\frac{2}{5} + \frac{11}{5}i.$$

$$\therefore Re\left(\frac{z_1 z_2}{\bar{z}_1}\right) = -\frac{2}{5}$$

(ii) $\frac{1}{z_1\bar{z}_1} = \frac{1}{(2-i)(2+i)} = \frac{1}{2^2-i^2}$

$$= \frac{1}{4+1} = \frac{1}{5}.$$

$\therefore Im\left(\frac{1}{z_1\bar{z}_2}\right) = 0$ #

Q. Find the modulus and argument of $\frac{1+2i}{1-3i}$.

Sol:- let $z = \frac{1+2i}{1-3i}$

$$= \frac{1+2i}{1-3i} \times \frac{1+3i}{1+3i} = \frac{(1+2i)(1+3i)}{1^2-(3i)^2}$$

$$= \frac{1+2i+3i+6i^2}{1+9} = \frac{-5+5i}{10}$$

$$= \frac{5(-1+i)}{10} = \frac{-1+i}{2} = \frac{-1}{2}+\frac{1}{2}i$$

$$|z| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \sqrt{\frac{2}{4}}$$

$$= \frac{1}{\sqrt{2}}.$$

$$Arg|z| = \tan^{-1}\frac{\frac{1}{2}}{-\frac{1}{2}} = \tan^{-1}(-1)$$

$$= \tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) = \frac{3\pi}{4}$$ #

Q.14. Find the real number $x$ and $y$ if $(x-iy)(3+5i)$ is the conjugate of $-6-24i$.

Sol: let $z_1 = (x-iy)(3+5i)$

and $z_2 = -6-24i$

According to question.

$z_1 = \overline{z_2}$

$\Rightarrow (x-iy)(3+5i) = -6+24i$

$\Rightarrow 3x - 5yi^2 - 3yi + 5xi = -6+24i$

$\Rightarrow 3x + 5y - 3yi + 5xi = -6+24i$

$\Rightarrow (3x+5y) + (5x-3y)i = -6+24i$

By comparing

$3x+5y = -6 \quad -(1)$

$5x-3y = 24 \quad -(11)$

Solving (1) and (11) we get

$x = 3, \ y = -3.$

Q15. Find the modulus of

$\frac{1+i}{1-i} - \frac{1-i}{1+i}$.

Sol: $\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{1^2 - i^2}$

$= \frac{(1+i^2+2i) - (1+i^2-2i)}{1+1}$

$= \frac{(1-1+2i) + (1-1-2i)}{1+1}$

$= \frac{2i+2i}{2} = \frac{4i}{2} = 2i$

$\therefore$ Modulus of $\left(\frac{1+i}{1-i} - \frac{1-i}{1+i}\right) = \sqrt{0+2^2}$

$= \sqrt{4} = 2$ Ans

Q16: If $(x+iy)^3 = u+iv$. Then show that:

$\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$

Sol: We have $(x+iy)^3 = u+iv$

$\Rightarrow x^3 + i^3y^3 + 3x^2iy + 3i^2y^2x = u+iv$

$\Rightarrow x^3 - iy^3 + 3xyi - 3y^2x = u+iv$

$\Rightarrow (x^3 - 3xy^2) + i(3x^2y - y^3) = u+iv$

Comparing

$u = x^3 - 3xy^2$

$u = x(x^2-3y^2)$

$\Rightarrow \frac{u}{x} = x^2 - 3y^2 \quad -(1)$

and $3x^2y - y^3 = v$

$y(3x^2-y^2) = v$

$\Rightarrow \frac{v}{y} = 3x^2 - y^2 \quad -(2)$

Adding (1) and (11)

$\frac{u}{x} + \frac{v}{y} = x^2 - 3y^2 + 3x^2 - y^2$

$= 4x^2 - 4y^2$

$\frac{u}{x} + \frac{v}{y} = 4(x^2-y^2)$, which is true.

Q17. If $\alpha$ and $\beta$ are different complex numbers with $|\beta| = 1$ then find $\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right|$.

Sol: let $\alpha = a+ib$ and $\beta = x+iy$

Given $|\beta| = 1$

$\therefore \sqrt{x^2+y^2} = 1 \Rightarrow x^2+y^2 = 1$

$$\left|\frac{\beta-\alpha}{1-\bar{\alpha}\beta}\right| = \left|\frac{(x+iy)-(a+ib)}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{x+iy-a-ib}{1-(a-ib)(x+iy)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+aiy-ibx-i^2by)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{1-(ax+by+ibx+iay)}\right|$$

$$= \left|\frac{(x-a)+i(y-b)}{(1-ax-by)+i(bx-ay)}\right|$$

$$= \frac{\sqrt{(x-a)^2+(y-b)^2}}{\sqrt{(1-ax-by)^2+(bx-ay)^2}}$$

$$= \frac{\sqrt{x^2+a^2-2ax+y^2+b^2-2by}}{\sqrt{1+a^2x^2+b^2y^2-2ax+2abxy-2by+b^2x^2+a^2y^2-2abxy}}$$

$$= \frac{\sqrt{(x^2+y^2)+(a^2+b^2-2ax-2by}}{\sqrt{1+a^2(x^2+y^2)+b^2(x^2+y^2)-2ax-2by}}$$

$[\because x^2+y^2=1]$.

$$= \frac{\sqrt{1+a^2+b^2-2ax-2by}}{\sqrt{1+a^2+b^2-2ax-2by}} = 1$$

Q18. Find the number of non-zero integral Solution of $|1-i|^x = 2^x$

Sol: We have : $|1-i|^x = 2^x$

$= \left(\sqrt{1^2+(-1)^2}\right|^x = 2^x$

$= \left|\sqrt{1+1}\right|^x = 2^x$

$\Rightarrow (\sqrt{2})^x = 2^x$

$\Rightarrow 2^{x/2} = 2^x \Rightarrow \frac{x}{2} = x$

$\Rightarrow \frac{x}{2} - x = 0 \Rightarrow \frac{x-2x}{2} = 0$

$\Rightarrow -\frac{x}{2} = 0 \Rightarrow -x = 0 \Rightarrow x = 0$

Hence the number of non-zero integral solution is zero.

Q19: If $(a+ib)(c+id)(e+if)(g+ih) = A+iB$ then prove that $(a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = A^2+B^2$

Solution:

$|(a+ib)(c+id)(e+if)(g+ih)| = |A+iB|$

$\Rightarrow |a+ib| \times |c+id| \times |e+if| \times |g+ih| = |A+iB|$

$= \sqrt{a^2+b^2} \times \sqrt{c^2+d^2} \times \sqrt{e^2+f^2} \times \sqrt{g^2+h^2} = \sqrt{A^2+B^2}$

Sq. both side

$= (a^2+b^2)(c^2+d^2)(e^2+f^2)(g^2+h^2) = (A^2+B^2)$.

Proved.

Q20: If $\left(\frac{1+i}{1-i}\right)^m = 1$, then find the least integral value of m.

Sol! $\frac{1+i}{1-i} \times \frac{1+i}{1+i} = \frac{(1+i)^2}{1^2-(i)^2}$

$= \frac{i^2+i^2+2i}{1+1} = \frac{1-i+2i}{2} = \frac{2i}{2} = i$

$\therefore \left(\frac{1+i}{1-i}\right)^m = i^m = 1$

$\Rightarrow (i^4)^{\frac{m}{4}} = 1$

$\Rightarrow \frac{m}{4} = 1 \Rightarrow m = 4$

$\Rightarrow$ m is a multiple of 4.

Hence least integral value of $m = 4$.