Formulae For

WAVE OPTICS

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CHAPTER FORMULAS & NOTES

Important Terms, Definitions & Formulae

Top Concepts

1. A wave front is the locus of points having the same phase of oscillation. Rays are the lines perpendicular to the wavefront, which show the direction of propagation of energy. The time taken for light to travel from one wavefront to another is the same along any ray.

2. Huygens' Principle.

According to Huygens'

- Each point on the given wave front (called primary wave front) (a) acts as a fresh source of new disturbance, called secondary wavelet, which travels in all directions with the velocity of light in the medium
- (b) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wave front,
- **3.** Huygens' Construction is based on the principle that every point of a wavefront is a source of secondary wavefront. The envelope of these wavefronts i.e., the surface tangent to all the secondary wavefront gives the new wavefront.

4. Refraction and Reflection of Plane Waves Using Huygens' Principle.

The law of reflection (i = r) and the Snell's law of refraction $\frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1} = \mu_{21}$

can be derived using the wave theory. (Here v_1 and v_2 are the speed of light in media 1 and 2 with refractive index μ_1 and μ_2 respectively).

The frequency v remains the same as light travels from one medium to another. The speed v of a wave is given by

$$v = \frac{\lambda}{T}$$

where λ is the wavelength of the wave and T (=1/v) is the period of oscillation.

5. Doppler effect is the shift in frequency of light when there is a relative motion between the source and the observer. The effect can be used to measure the speed of an approaching or receding object.

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for the source moving away from the observer, $v < v_0$ and for the source moving towards the observer, $v > v_0$. The change in frequency is given as

$$\Delta v = v - v_0 \square - \frac{v}{c} v_0$$
 where we are using the approximation $v \square c$.

So, finally, $\frac{\Delta v}{v_0} = -\frac{v}{c}$

6. Coherent and Incoherent Addition of Waves. Two sources are coherent if they have the same frequency and a stable phase difference. In this case, the total intensity I is not just the sum of individual intensities I_1 and I_2 due to the two sources but includes an interference term:

$$I = I_1 + I_2 + 2k.E_1.E_2$$

where E_1 and E_2 are the electric fields at a point due to the sources. The interference term averaged over many cycles is zero if

- (a) the sources have different frequencies; or
- (b) the sources have the same frequency but no stable phase difference.

For such coherent sources, $I = I_1 + I_2$.

According to the superposition principle when two or more wave motions traveling through a medium superimpose one another, a new wave is formed in which resultant displacements due to the individual waves at that instant.

The average of the *total* intensity will be

$$\overline{I} = \overline{I_1} + \overline{I_2} + 2\sqrt{\left(\overline{I_1}\right)\left(\overline{I_2}\right)} \cos \Phi$$

where ϕ is the inherent phase difference between the two superimposing waves.

The significance is that the intensity due to two sources of light *is not equal* to the sum of intensities due to each of them. The resultant intensity depends on the relative location of the point from the two sources, since changing it changes the path difference as we go from one point to another. As a result, the resulting intensity will vary between maximum and minimum values, determined by the maximum and minimum values of the cosine function. These will be

$$\begin{split} \overline{I}_{MAX} &= \overline{I_1} + \overline{I_2} + 2\sqrt{\left(\overline{I_1}\right)\left(\overline{I_2}\right)} = \left(\sqrt{\overline{I_1}} + \sqrt{\overline{I_2}}\right)^2 \\ \overline{I}_{MIN} &= \overline{I_1} + \overline{I_2} - 2\sqrt{\left(\overline{I_1}\right)\left(\overline{I_2}\right)} = \left(\sqrt{\overline{I_1}} - \sqrt{\overline{I_2}}\right)^2 \end{split}$$

7. Young's experiment, two parallel and very close slits S₁ and S₂ (illuminated by another narrow slit) behave like two coherent sources and produce on a screen a pattern of dark and bright bands – interference fringes. For a point P on the screen, the path difference

$$S_2P - S_2P = \frac{y_1d}{D_1}$$

where d is the separation between two slits, D_1 is the distance between the slits and the screen and y_1 is the distance of the point of *P* from the central fringe.

For constructive interference (bright band), the path difference must be an integer multiple of λ , i.e.,

$$\frac{y_1d}{D_1} = n\lambda$$
 or $y_1 = n\frac{D_1\lambda}{d}$

The separation Δy_1 between adjacent bright (or dark) fringes is.

$$\Box \mathbf{y}_1 = \frac{\mathbf{D}_1 \lambda}{\mathbf{d}}$$

using which $\boldsymbol{\lambda}$ can be measured.

8. Diffraction refers to light spreading out from narrow holes and slits, and bending around corners and obstacles. The single-slit diffraction pattern shows the central maximum (at $\theta = 0$), zero intensity at angular separation $\theta = \pm (n + \frac{1}{2})\lambda$... $(n \neq 0)$.

Different parts of the wavefront at the slit act as secondary sources: diffraction pattern is the result of interference of waves from these sources.

The intensity plot looks as follows, with there being a bright central maximum, followed by smaller intensity *secondary maxima*, with there being points of zero intensity in between, whenever $d \sin \theta = n\lambda$, $n \neq 0$



9. Emission, absorption and scattering are three processes by which matter interacts with radiation.

In emission, an accelerated charge radiates and loses energy. In absorption, the charge gains energy at the expense of the electromagnetic wave.

In scattering, the charge accelerated by incident electromagnetic wave radiates in all direction.

10. Polarization specifies the manner in which electric field *E* oscillates in the plane transverse to the direction of propagation of light. If *E* oscillates back and forth in a straight line, the wave is said to be

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linearly polarized. If the direction of E changes irregularly the wave is unpolarized.

When light passes through a single polaroid P_1 light intensity is reduced to half, independent of the orientation of P_1 . When a second Polaroid P_2 is also included, at one specific orientation wrt P1, the net transmitted intensity is reduced to zero but is transmitted fully when P_1 is turned 90° from that orientation. This happens because the transmitted polarization by a polaroid is the component of *E* parallel to its axis.

Unpolarized sunlight scattered by the atmosphere or reflected from a medium gets (partially) polarized.

Linearly Polarized light passing through some substances like sugar solution undergoes a rotation of its direction of polarization, proportional to the length of the medium traversed and the concentration to the substance. This effect is known as optical activity.

11. Brewster's Law: When an incident light is incident at the polarizing angle, the reflected & the refracted rays are perpendicular to each other. The polarizing angle, also called as Brewster's angle, is given by $tan\theta_p = \mu$

this expression is also called Brewster's law.

12. Polarization by scattering: Light is scattered when it meets a particle of similar size to its own wavelength. For e.g. scattering of sunlight by dust particles.

Rayleigh showed that the scattering of light is proportional to the fourth

power of the frequency of the light or varies as $\frac{1}{\lambda^4}$ where λ is the

wavelength of light incident on the air molecules of size d where d $<< \lambda$. Hence blue light is scattered more than red. This explains the blue colour of the sky.

TOP Formulae

1. Snell's law of refraction:

 $_{1}\mu_{2} = \frac{c_{1}}{c_{2}} = \frac{\text{speed of light in first medium}}{\text{speed of light in second medium}}$

2. Relation between phase difference & path difference:

$$\Delta \phi = \frac{2\pi}{\lambda}. \ \Delta x$$

where $\Delta \phi$ is the phase difference & Δx is the path difference 3. Young's double slit interference experiment:

Fringe width: $W = \frac{D\lambda}{d}$

where D is the distance between the slits & the screen d is the distance between the two slits $% \left({{{\rm{D}}_{\rm{B}}}} \right) = {{\rm{D}}_{\rm{B}}} \left({{{\rm{D}}_{\rm{B}}}} \right) = {{{\rm{D}}_{\rm{B}}} \left({{{\rm{D}}_{\rm{B}}}} \right) = {{{\rm{D}}_{\rm{B}}}} \left({{{\rm{D}}_{\rm{B}}}} \right) = {{{\rm{D}}_{\rm{B}}} \left($

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Constructive interference:

Phase difference : $\Delta \phi = 2\pi n$ where n is an integer Path difference: $\Delta x = n\lambda$, where n is an integer

Destructive interference:

Phase difference : $\Delta \phi = \left(n + \frac{1}{2}\right)2\pi$ where n is an integer Path difference: $\Delta x = \left(n + \frac{1}{2}\right)\lambda$, where n is an integer

4. Diffraction due to single slit:

Angular spread of the central maxima = $\frac{2\lambda}{d}$

Width of the central maxima: $\frac{2\lambda D}{d}$

where D is the distance of the slit from the screen d is the slit width

Condition for the minima on the either side of the central maxima: $d \sin \theta = n\lambda$, where n = 1, 2, 3, ...

5. Intensity of the light due to polarization: $I = I_0 \cos^2 \theta$ where I is the intensity of light after polarization I_0 is the original intensity θ is the angle between the axis of the analyzer & the polarizer

Brewster's Law:

 $\mu = tan\theta_p$ where θ_p is the polarizing angle, that is, the angle of incidence at which the angle of refraction in the second medium is right angle