SA

14.29 Find the time period of mass M when displaced from its equilibrium positon and then released for the system shown in Fig 14.10.

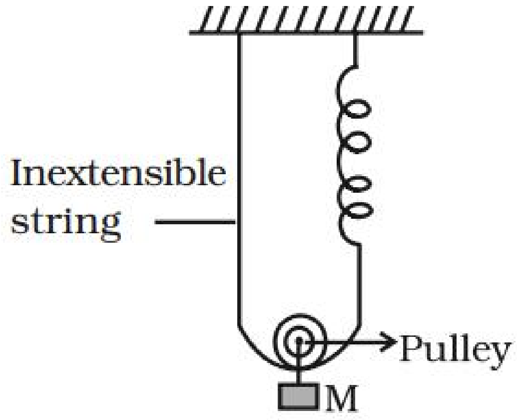


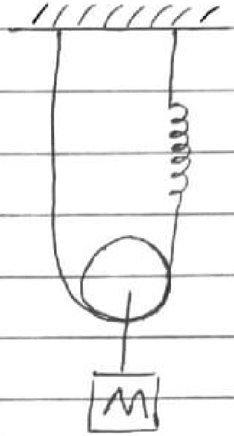
Fig. 14.10

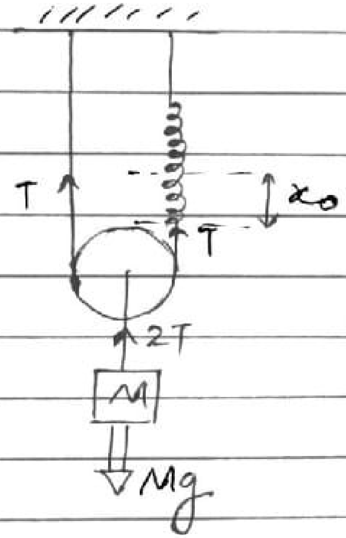
NCERT EXEMPLAR

SOLUTION:

FOR finding Time Period:

Step 1: Analyse equilibrium point.





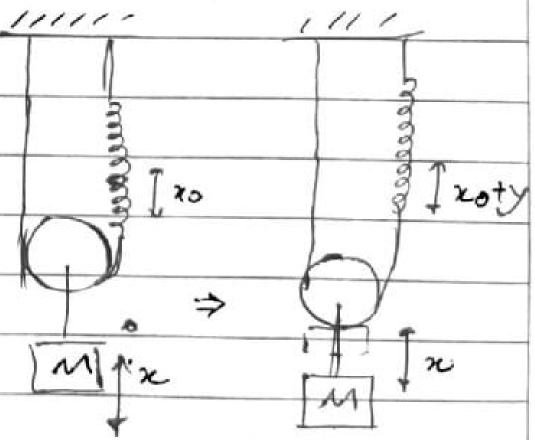
so, from FBD of M-

$2T = Mg$ and $T = kx_0$

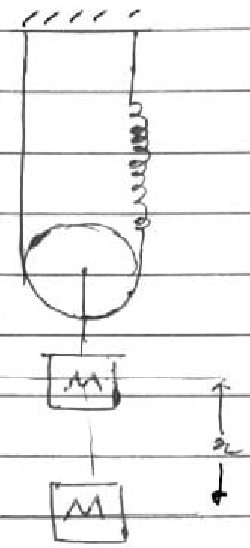
$\Rightarrow \boxed{2kx_0 = Mg}$

Step 2: Displace the mass by 'x' and find the restoring force on the mass

~~Push~~ Displacing 'm' by 'x' would not extend the spring by 'x'. The extension would be different. let's suppose it 'y'



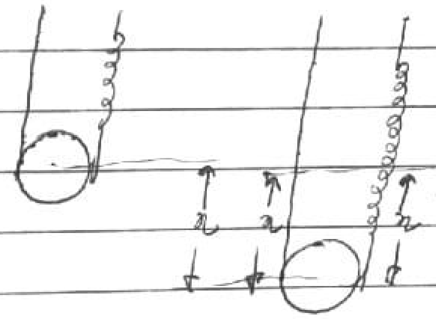
for finding 'y'



Since the mass is pushed down by 'x', the pulley will come down by 'x', increasing the length of (wire + spring) by

$x + x$

↓ Two ~~sides~~ sides



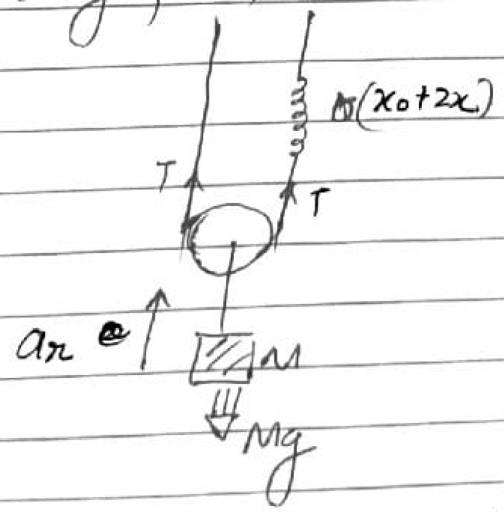
Increase in length

$= x + x = 2x$

Now since string cannot extend.

So extension in spring is '$2x$'.

Now restoring force,



so, $T = k(x_0 + 2x)$

and

$m a_r = 2T - Mg$

$m a_r = \cancel{2kx_0} + 4kx - \cancel{Mg}$

$m a_r = 4kx$ $\boxed{2kx_0 = Mg}$

so, now $a_r = -a$

a_r = Restoring Force

So, $-ma = 4kx$

$$a = \frac{-4k}{m}x \quad \text{so} \quad \omega^2 = \frac{4k}{m}$$

$$\omega = \sqrt{\frac{4k}{m}}$$

Now, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$