3.

IMPORTANT CONCEPT ⇒

- we can solve the system of linear eqⁿs in two ways.

(i)
$$AX = B$$
$$X = A^{-1}B$$

where $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

and if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

then $(\text{Adj } A) = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$

Here $M_{11} = \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix}$; $M_{12} = \begin{bmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{bmatrix}$; $M_{13} = \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$

$M_{21} = \begin{bmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{bmatrix}$; $M_{22} = \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$; $M_{23} = \begin{bmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{bmatrix}$

$M_{31} = \begin{bmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{bmatrix}$; $M_{32} = \begin{bmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{bmatrix}$; $M_{33} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

$P = \begin{bmatrix} +M_{11} & -M_{12} & +M_{13} \\ -M_{21} & +M_{22} & -M_{23} \\ +M_{31} & -M_{32} & +M_{33} \end{bmatrix}$ ⇒ $(\text{adj } A) = [P]^T$

(ii) Row-Reduced Echlon Method ⇒

Here we try to make as many zeroes as we can in a row by applying row elementary operations. And after that

if Rank(A) = Rank(A|B) ; System has solutions

Rank(A) ≠ Rank(A|B) : System has no solution

(A|B) ⇒ augmented matrix.

Rank(A) ⇒ number of linearly independent rows or columns in the matrix. (if matrix is in row echelon form)

if A is of n×n order and $|A| \neq 0$ ⇒ Rank(A) = n

if A is of n×n order and $|A| = 0$ ⇒ Rank(A) < n