

Q- $f(x) = \frac{1+x}{1-x}$. If A is a matrix for which $A^3 = O$, then $f(A)$ is

- a) $I + A + A^2$
- b) $I + 2A + 2A^2$
- c) $I - A - A^2$
- d) none of these

$$\left\{ \begin{array}{l} \cancel{f(A)} \\ \cancel{\frac{I+A}{I-A}} \end{array} \right\}$$

Soln-

$$(1-x)f(x) = 1+x$$

$$\Rightarrow (I-A)f(A) = I+A$$

$$f(A) = (I+A)(I-A)^{-1}$$

$$A^3 = O \quad (\text{given})$$

$$A^3 + I = I$$

$$I = I - A^3$$

or $I - A^3 = I$

$$(I-A)(I+A+A^2) = I$$

$\left. \begin{array}{l} \star \\ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \\ \text{can be used} \\ \text{in matrix if } AB = BA \end{array} \right\}$

$$\left\{ \begin{array}{l} A \cdot B = I \\ \Rightarrow A^{-1} = B \end{array} \right\}$$

hence, $(I-A)^{-1} = I + A + A^2$

$$f(A) = (I+A)(I+A+A^2)$$

$$= I + A + A^2 + A + A^2 + A^3$$

$$f(A) = I + 2A + 2A^2$$

$\{ A^3 = O, \text{ given} \}$

b) is correct.