For Problems 4–6

If A and B are two square matrices of order 3×3 which satisfy $AB = A$ and $BA = B$, then

4. Which of the following is true?

   a. If matrix A is singular then matrix B is non-singular.

   b. If matrix A is non-singular then matrix B is singular.

   c. If matrix A is singular then matrix B is also singular.

   d. Cannot say anything.

5. $(A + B)^7$ is equal to

   a. $7(A + B)$ **b.** $7 \cdot I_{3 \times 3}$

   c. $64\,(A + B)$ **d.** $128\,I$

6. $(A + I)^5$ is equal to (where I is identity matrix)

   a. $I + 60I$ **b.** $I + 16A$

   c. $I + 31A$ **d.** none of these

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$AB = A$

$BA = B$

(4) $AB = A$

$|AB| = |A|$ $\left\{ \begin{array}{l} \text{formula used-} \\ |A \cdot B| = |A| \cdot |B| \end{array} \right\}$

$|A| \cdot |B| = |A|$

$|A| \{ |B| - 1 \} = 0$

Now, either $|A| = 0$ or $|B| = 1$ $\rightarrow$ (1)

Similarly, $|BA| = |B|$ $\left\{ \begin{array}{l} BA = B \\ \text{is given.} \end{array} \right\}$

$\Rightarrow |B| = 0$ or $|A| = 1$ $\rightarrow$ (2)

from (1) & (11)

If $|A| = 1$ then $|B| = 1$

If $|A| = 0$ then $|B| = 0$

(c) is correct.

(5) $AB = A$

$BA = B$

$BA = B$

$\boxed{BAB = B} \rightarrow$ (1) $\{A = AB$, Given$\}$

$BA = B$

$(BA) \cdot B = (B) \cdot B$

$BAB = B^2$

$B = B^2$ (from (1))

$\boxed{B^2 = B}$

Similarly, $\boxed{A^2 = A}$

Now, $(A+B)^2 = A^2 + B^2 + AB + BA$

$(A+B)^2 = A + B + A + B$

$\boxed{(A+B)^2 = 2(A+B)} \longrightarrow (2)$

$(A+B)^3 = 2(A+B)^2$

$(A+B)^3 = 2^2 (A+B)$ (from (2))

Thus, $(A+B)^7 = 2^6 (A+B)$

$= \underline{64(A+B)}$ Ans.

(c) is correct.

(6) $(A+I)^5 = I + 5A + 10A^2 + 10A^3 + 5A^4 + A^5$

{Expansion - simply using Binomial Theorem because $A \cdot I = I \cdot A$ }

$\left\{ \begin{array}{l} A \cdot I = I \cdot A = A \\ I^n = I \quad \text{, where } n \text{ is a natural number} \end{array} \right\}$

As proved in Q-5, $A^2 = A$

$$A^3 = A^2 \cdot A$$
$$= A \cdot A$$
$$= A^2$$
$$A^3 = A$$

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Similarly, $A^4 = A$

& $A^5 = A$

Now,

$(A + I)^5 = I + 5A + 10A + 10A + 5A + A$

$= I + 31A$

(c) is correct.

☆ <u>Note</u>:- If A & B are square matrices of same order.

If $A \cdot B = B \cdot A$, $(A+B)^n$ can be expanded using Binomeal Theorem.