1 Intersection of Line and Circle

Suppose that a general line equation is $y = mx + c$ and a circle with origin as its center is $x^2 + y^2 = r^2$. To know whether line intersects circle or not, simply put value of y from line in circle. This gives a simple quadratic equation in x. Now one can consider different cases of existence of roots of this quadratic equation to situation of Line and Circle.

$$x^2 + (mx + c)^2 = r^2$$

Three cases can be analysed,

case-1: If both roots are real and different then we have two intersection points.

case-2: If both roots are same and real then we have one intersection points and the line is a tangent of the circle.

case-3: If one root is imaginary then second one will be its complex-conjugate then we have no intersection points.
Try to visualize these scenarios.

2 Intercept made by Circle on Axes

Consider general form of a circle $x^2 + y^2 + 2gx + 2fy + c = 0$. To have a visual clarity of concept have a look at below diagram,

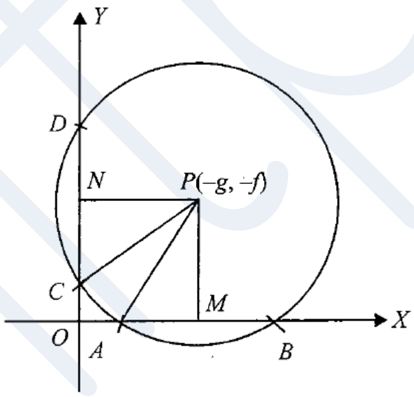

Figure 1: Intercept made by Circle on Axes

From diagram it is clear that,

$$PM = |g|$$
$$PN = |f|$$
$$AP = CP = r = \sqrt{g^2 + f^2 - c}$$

From triangles CPN and APM we have intercept terms as,

$$AB = 2AM = 2\sqrt{f^2 - c}$$
$$CD = 2CN = 2\sqrt{g^2 - c}$$

Thing to note is that make a clear diagram of scenario that is under consideration and proceed from there.

NOTE:

1. Since intercepts are square roots, they are always positive.
2. When circle touches either axis, then that axis has intercept to be zero since $f^2 = c$ or $g^2 = c$.
3. When circle touches both axis then both intercepts are zero since $f^2 = c$ and $g^2 = c$.