Tips and Tricks to for circle tangent and normal:

Tangent and Normal Equation: A General Tip

Rather than trying to remember tangent and normal equation for specific curves like circle, one can simply learn to get such equations for any curve at a point; given that it is at least once differentiable. Practice this one on paper to understand the concept.

We know that the equation of the straight line that passes through the point (x_0, y_0) with finite slope "m" is given as

$y - y_0 = m (x - x_0)$

It is noted that the slope of the tangent line to the curve f(x)=y at the point (x_0, y_0) is given by

$\frac{dy}{dx}\big]_{(x_0,y_0)} (= f'(x_0))$

Therefore, the equation of the tangent (x_0, y_0) to the curve y=f(x) is

$$y - y_0 = f'(x_0)(x - x_0)$$

Also, we know that normal is the perpendicular to the tangent line. Hence, the slope of the normal to the curve f(x)=y at the point (x_0, y_0) is given by $-1/f'(x_0)$, if $f'(x_0) \neq 0$.

Hence, the equation of the normal to the curve y=f(x) at the point (x_0, y_0) is given as:

$y-y_0 = [-1/f'(x_0)] (x-x_0)$

The above expression can also be written as

$$(y-y_0) f'(x_0) + (x-x_0) = 0$$

Circle is a specific curve. Hence simply put f(x) to be circle equation, and proceed from there. Here is a solved example.

Example 3: Find the equation of normal to the circle $2x^2 + 2y^2 - 2x - 5y + 3 = 0$ at (1, 1).

Solution:

The centre of the circle is (1/2, 5/4)

Normal to circle at point (1, 1) is line passing through the points (1, 1) and (1/2, 5/4) which is $x + 2y = 3$.

Trick:

Prof, also talked about length of tengent from a point. Best tip is to remember the diagram shown on right side. Also remember the point that length from point P is square root of general form of circle with coordinates of point P, replaced in place.

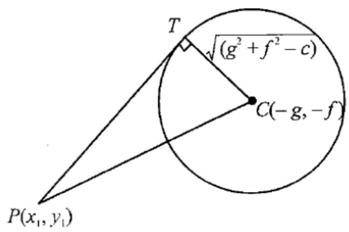
Length of PC,

$$PT = \sqrt{(PC)^2 - (CP)^2}$$

Put value of PC and CP, we get after solving

$$PT = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

Put point value in general form of circle, and take its sq. root to get length of tangent from a point.



NOTE: For detailed proof of formulas, please read class notes.

Useful forms for tangets on circle:

VARIOUS FORMS OF EQUATIONS OF TANGENTS IN CIRCLE

$$x^2 + y^2 = a^2$$

	Point	Equation of Tangent
Point Form	(x_1, y_1)	$xx_1 + yy_1 = a^2$
Slope Form	$\left(\mp\frac{ma}{\sqrt{1+m^2}}, \pm\frac{a}{\sqrt{1+m^2}}\right)$	$y = mx \pm a\sqrt{1+m^2}$
Parametric Form	$(a\cos\theta, a\sin\theta)$	$x\cos\theta + y\sin\theta = a$