Practice Questions

Q1. Show that the point (x, y) given by $x = \frac{2at}{1+t^2}$ and $y = \frac{a(1-t^2)}{1+t^2}$ lies on a circle for all real values of t such that $-1 < t < 1$ where a is any given real numbers.

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Q2. If the lines $3x - 4y + 4 = 0$ and $6x - 8y - 7 = 0$ are tangents to a circle, then find the radius of the circle.
Hint: Distance between given parallel lines gives the diameter of the circle

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Q3. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.
Hint: Let a be the radius of the circle, then (- a, - a) will be centre and perpendicular distance from the centre to the given line gives the radius of the circle.

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Q4. If the line $y = \sqrt{3}x + k$ touches the circle $x^2 + y^2 = 16$, then find the value of k.
Hint: Equate perpendicular distance from the centre of the circle to its radius.

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Solution and Hints

S1. Given point is,

$$x = \frac{2at}{1+t^2}, y = \frac{a(1-t^2)}{1+t^2}$$

Put its coordinates in $x^2 + y^2$ to see what it gives,

$$\begin{aligned} x^2 + y^2 &= \left(\frac{2at}{1+t^2}\right)^2 + \left(\frac{a(1-t^2)}{1+t^2}\right)^2 \\ &= \frac{4a^2t^2 + a^2(1-t^2)^2}{(1+t^2)^2} \\ &= \frac{a^2(1+t^4+2t^2)}{(1+t^2)^2} \\ &= \frac{a^2(1+t^2)^2}{(1+t^2)^2} \\ &= a^2 \end{aligned}$$

Now compare it with center-radius form $(x-h)^2 + (y-k)^2 = r^2$ and we get,

$$\begin{aligned} center &= (0,0) \\ radius &= a \end{aligned}$$

S2. Just by looking at st. line equations we can see that they are —— lines. They touch circle on ends of diameter line. So distance bwtween these tangents should give us diameter of the circle. Equate both lines coefficients,

$$\begin{aligned} 3x - 4y + 4 = 0 \\ 3x - 4y - 3.5 = 0 \end{aligned}$$

Recall from st. line chapter that distance bwtween two —— lines is,

$$dist = \left| \frac{c-d}{\sqrt{a^2+b^2}} \right|$$

Compare given st. lines with $ax + by + c = 0$ and $ax + by + d = 0$ and we get,

$$dist = \left| \frac{4 - (-3.5)}{\sqrt{3^2 + (-4)^2}} \right|$$

By solving above, distance comes out = 1.5. Since this is the diameter of the circle. Radius is,

$$r = \frac{1.5}{2} = 0.75 \text{ units}$$

S3. Since circle touches both axes in 3rd quadrant, its center can be assumed to be (-a,-a). We need to find **a**.

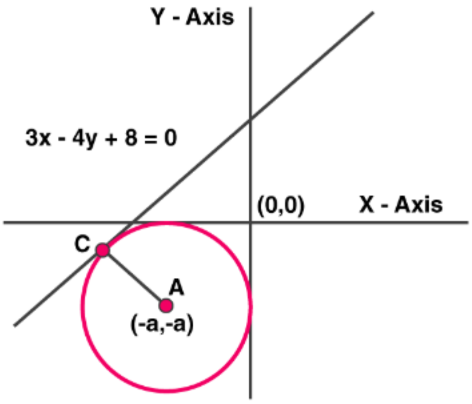

Figure 1: Circle in third quadrant

Also recall from st. lines chapter that parpendicular distance of st. line($ax + by + c = 0$) from a point (x_1, y_1) is given by,

$$= \left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$$

Since radius(=a) is the distance, we can compare both to get value of a.

$$a = \left| \frac{3(-a) - 4(-a) + 8}{\sqrt{3^2 + (-4)^2}} \right|$$

$$= \left| \frac{8 + a}{\sqrt{25}} \right|$$

$$= \frac{8 + a}{5}$$

$$5a = a + 8$$

$$a = 2$$

Now put values in center radius form of circle,

$$(x - (-2))^2 + (y - (-2))^2 = a^2$$

General form after expanding is,

$$x^2 + y^2 + 4x + 4y + 4 = 0$$

S4. This problem uses similar concept to Prob-3. We need to compute tangent line's distance from center and equate it with known radius to get value of k. Comparing circle equation with center-radius form we get, center $= (0, 0)$ and radius $= 4$. After converting line equation in $ax + by + c = 0$ form and using similar formula as prob-3,

$$\sqrt{3}x - y + k = 0$$

Compare distance from formula with radius,

$$4 = \left| \frac{\sqrt{3}(0) - 1(0) + k}{\sqrt{\sqrt{3}^2 + (-1)^2}} \right|$$

$$4 = \frac{k}{\sqrt{4}}$$

$$k = (4).(2) = 8$$

Hence the value of $k = 8$.