classmate

Date
Page

Lec-3 :

1. $P(A)$, where A is an event corresponding to a random experiment E whose sample space is Ω is $\frac{\#A}{\#\Omega}$

(# means cardinality)

2. • $P(A \cup B) = P(A) + P(B)$
if A and B are disjoint

• $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
if A and B are not disjoint

3. If A and B are independent, then

$$P(A \cap B) = P(A) \times P(B)$$

1. If $X, Y, Z \geq 0$ and $X + Y + Z = 10$, then ~~no of possible solutions is~~.
What is the probability that X is an odd number?

Solution:

First, we find the no. of possible solutions.

No. of solutions $= {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$ [Refer to previous lectures]

X should be odd number

$\Rightarrow$ X can be 1, 3, 5, 7 or 9

classmate

Date

Page

Let $X = 1$,

then $Y + Z = 9$

Find No. of ~~ut~~ ways in which 9 can be split into Y and Z.

(0,9), (1,8), (2,7), (3,6), (4,5), (5,4),
(6,3), (7,2), (8,1), (9,0)

There are 10 ways or possibilities when $X = 1$.

Now, Let $X = 3$

then $Y + Z = 7$

If you use the same method, you will find that there are 8 ways in which

$Y + Z = 7$

Do the same for $X = 5$, $X = 7$ and $X = 9$

You will find that

	No. of ways
$X = 1$	10
$X = 3$	8
$X = 5$	6
$X = 7$	4
$X = 9$	2
	30

So, total no. of ways in which X will be odd $= 30$

$$\text{Probability} = \frac{30}{66} = \frac{5}{11}$$

2. Suppose a man is standing on the x-axis at the origin. At each step, he goes either to the right or to the left each with probability $= \frac{1}{2}$.

What is the probability that after 6 steps he will be at a distance 2 from the origin?

Solution

The man needs to be at a distance of 2 from the origin.

So, he needs to be either at +2 or −2. after 6 steps.

To go to +2, he needs to go 4 steps right and 2 steps left in any order.

In any order means that for eg. he can go one step right then one step left and 3 steps right and one step left. ~~and in any other way etc.~~

He can go in any order as long as he is taking 4 steps in total to the right and 2 steps in total to the left.

Probability that he will take 4 steps left and 2 steps right $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^6$

because probability of taking a step in any direction is half and he is taking 6 steps

Now, to get to -2, he must take 4 steps left and 2 steps right in any order.

Probability that he will go to $-2 = \left(\frac{1}{2}\right)^6$

So, total probability that he will be at distance 2 from origin $= \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^6 = \left(\frac{1}{2}\right)^6 \times 2$

$$= \left(\frac{1}{2}\right)^5$$

We added because he had to go to -2 "OR" $+2$. So, whenever there is OR, we add the probabilities.

Q3. Suppose A, B, C are three events corresponding to a random experiment E, such that,

i) $P(A \cup B \cup C) = 1$

ii) A, B and C are equally likely.

iii) $A \cap B$, $B \cap C$, $A \cap C$ are also equally likely

iv) $P(A \cap B) = \frac{P(A)}{2}$

and $P(A \cap B \cap C) = \frac{P(A \cap B)}{2}$

Find the probability of $A \cap B \cap C^c$

solution:

A, B and C are equally likely,

$\Rightarrow \quad P(A) = P(B) = P(C) \quad - (1)$

Also $A \cap B$, $B \cap C$, $C \cap A$ are equally likely,

$\Rightarrow \quad P(A \cap B) = P(B \cap C) = P(C \cap A) \quad - (2)$

We know this formula: (If you don't know, remember it)

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C) \quad - (3)$$

Using (1) & (2) in (3)

$$P(A \cup B \cup C) = 3P(A) - 3P(A \cap B) + P(A \cap B \cap C)$$

Given, $P(A \cup B \cup C) = 1$

$P(A \cap B) = \frac{P(A)}{2}$

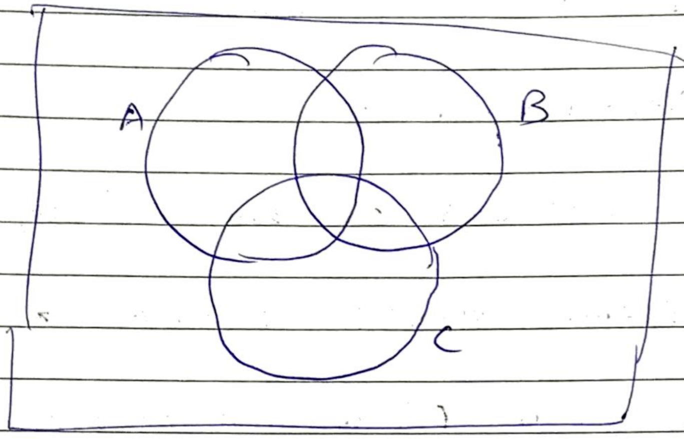
$$P(A \cap B \cap C) = \frac{P(A \cap B)}{2} = \frac{P(A)}{4}$$

$$1 = 3P(A) - \frac{3}{2}P(A) + \frac{P(A)}{4}$$

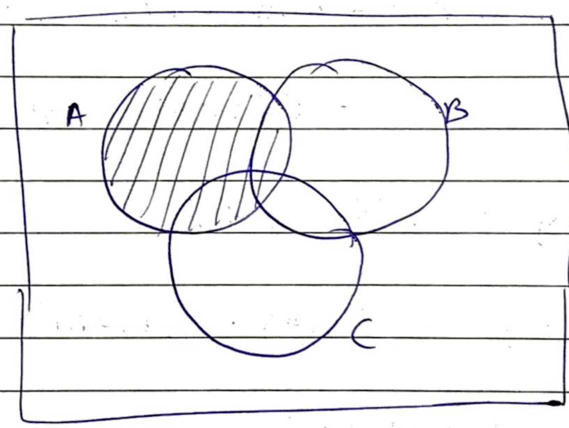
$$1 = \left(\frac{\cancel{4}12 - 6 + 1}{4}\right) P(A)$$

$$1 = \frac{7}{4} P(A) \Rightarrow P(A) = \frac{4}{7} = P(B) = P(C)$$

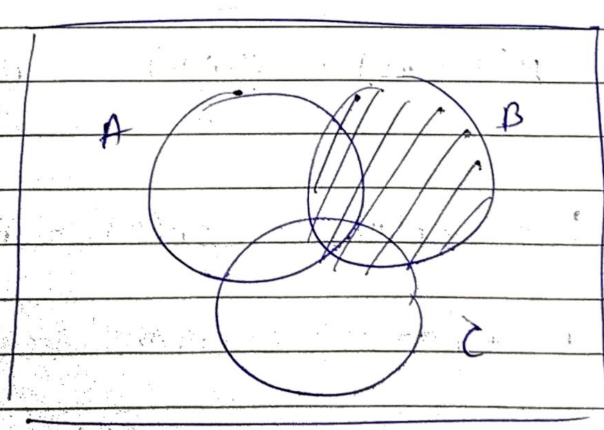
Now, let's make a Venn Diagram,



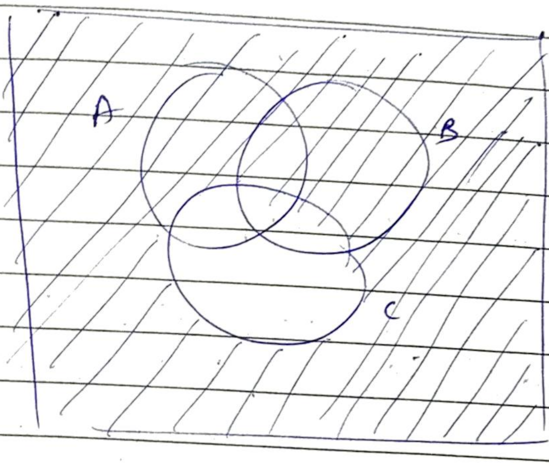
Mark A



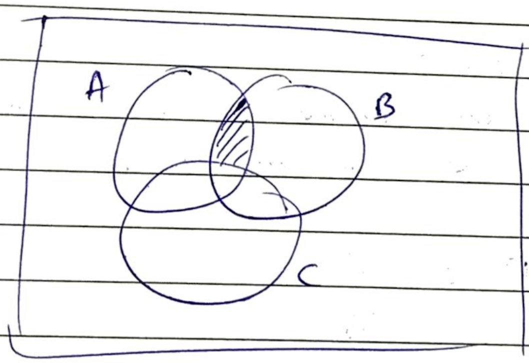
Mark ~~6~~ B



Mark C^c



Now, to mark $A \cap B \cap C'$, see ~~all pa~~ what parts are commonly shaded in A, B & C^c



So, $P(A \cap B \cap C^c) = P(A \cap B) - P(A \cap B \cap C)$

$$= \frac{P(A)}{2} - \frac{P(A)}{4} = \frac{P(A)}{4} = \frac{1}{4} \times \frac{4}{7}$$

$$= \frac{1}{7}$$

classmate
Date
Page

Q4. Suppose a pair of dice is rolled. The dice is fair i.e. P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6. What is the probability that the sum is 8 or you get even numbers on both the dice?

Solution: Let us ~~make~~ write the sample space.

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

~~P(sum is 8) =~~

Circle the numbers whose sum is 8 or both are even.

$$\text{So, } P = \frac{11}{36}$$

Q5. Suppose A and B are playing the final of a tournament against each other. The final may be best of 3 matches or best of 5 matches. The probability that A wins a match against B is 0.4.

But A has the advantage that he can choose whether it will be best of three or best of 5.

Which one should A choose?

~~Solution:~~

~~To win best of 3 matches, A must win atleast 2 matches.~~

~~Denote win by W and loss by L.~~

~~So, to win best of 3,~~

Solution:

To win best of 3,

A must either win both matches

or win one of the first two and win the second.

That means either WW or WLW or LWW

So, P(A winning best of 3) = P(WW) + P(WLW) + P(LWW)

$= 0.4 \times 0.4 + 0.4 \times 0.6 \times 0.4 + 0.6 \times 0.4 \times 0.4$

Player must win both matches means he must win first match 'AND' second match. 'AND' means you should multiply the probabilities

'OR' means add

$= 0.352$

To win best of 5 matches,

WWW or WWLW or WLWW
or LWWW or LLWWW or WWLLW
or WLWLW or LWLWW or WLLWW
or LWWLW

P(A wins best of 5)

$= 0.4 \times 0.4 \times 0.4 + (0.4 \times 0.4 \times 0.6 \times 0.4) \times 3$

$+ (0.6 \times 0.6 \times 0.4 \times 0.4 \times 0.4) \times 6$

$= 0.31744$

So, A has greater chance of winning if he chooses to play best of 3.

Q6. Suppose 5 candidates A, B, C, D and E are waiting to appear in an interview. It is known that the interview board will call them randomly in any order.

a) Find the probability that A is called before B.

b) Find the probability that A comes before B and B is called before C.

c) Find the probability that B is called just after A.

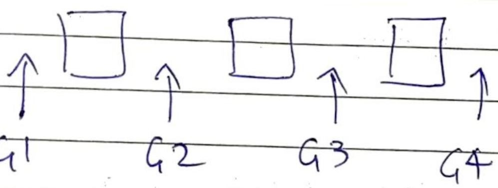
Solution:

In other words you have to arrange five people in a line.

No. of ways you can arrange 5 people in a line $= 5!$
$= 120.$

a) A must come before B

First we will arange C, D and E in a line in all possible ways.
So, they will occupy three places



G1, G2, G3 & G4 are gaps where we can place A and B.

Now, select two gaps from the four gaps and place A in the first gap and B in the second. or place A and B in the same gap by selecting one gap.
So, no. of ways in which A will come before B

$$= \underbrace{3!}_{\text{arranging C, D, E in a line}} \times \left({}^4C_2 + {}^4C_1 \right) = 6 \times (6+4) = 60$$

selecting two gaps from four

putting AB in one of the four gaps

Probability $= \frac{60}{120} = \frac{1}{2}$

NOTE: No. of ways in which n no. of objects can be arranged such that k ~~of~~ out of the ~~n~~ object must be in a specific order

$= \frac{n!}{k!}$

eg. In this question A must come before B in the selection of A, B, C, D, E

So, $n = 5$

and $k = 2$ because two people A and B must come in an order

So, no. of ways $= \frac{5!}{2!} = 60$

b) A must come before B and B must come before C.

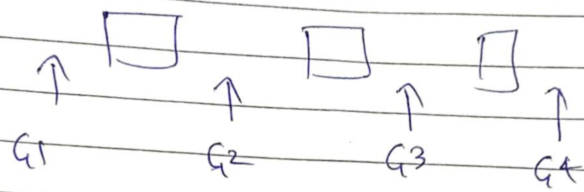
We can use our formula given above. ~~and~~

Here there are five objects and three of them need to be in a specific order.

So, no. of ways $= \frac{5!}{3!} = 20$

c) B is called just after A.

First arrange C, D, E in a line just like in part a)



Now, we want A first then B just after it.

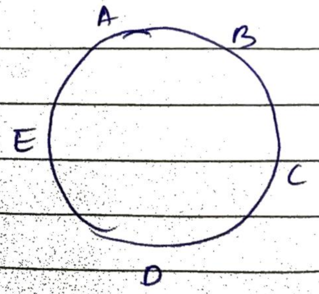
So, we select any one of the gaps and put AB in it.

So; no. of ways = $\underbrace{3!}_{\text{arranging C, D, E}} \times \underbrace{{}^4C_1}_{\text{choosing one out of four gaps.}} = 24$

Q7. Suppose 5 persons A, B, C, D and E are sitting in a circular table.
You have hats of three colours : W, R and G.

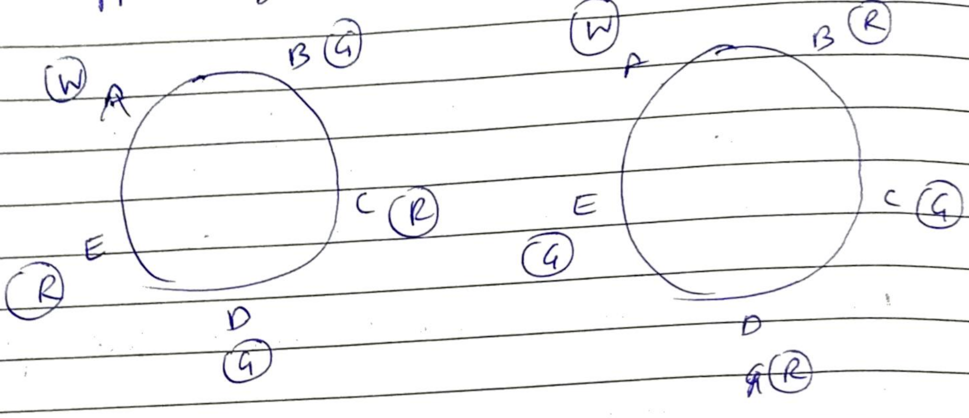
In how many ways can you give one hat to each person, ∋ no two consecutive persons have hats of same color?

Solution:



~~Suppose A gets W~~

Suppose A gets W, then B will have G or R



So, if A gets W, there are two possible arrangements.
Similary " " " G, "
and " " " R, "

And in all three cases only A has a ~~colour~~ unique colour.

So, total cases where A has unique color $= 2+2+2$
$= 6$

Any person can have a unique colour.

So, total ~~ca~~ possible arrangements $= 6 \times 5$
$= 30$