

Question. In $\triangle ABC$, AD , BE and CF are three medians.

Then the ratio

$$\frac{AD^2 + BE^2 + CF^2}{AB^2 + BC^2 + AC^2} \text{ is ?}$$

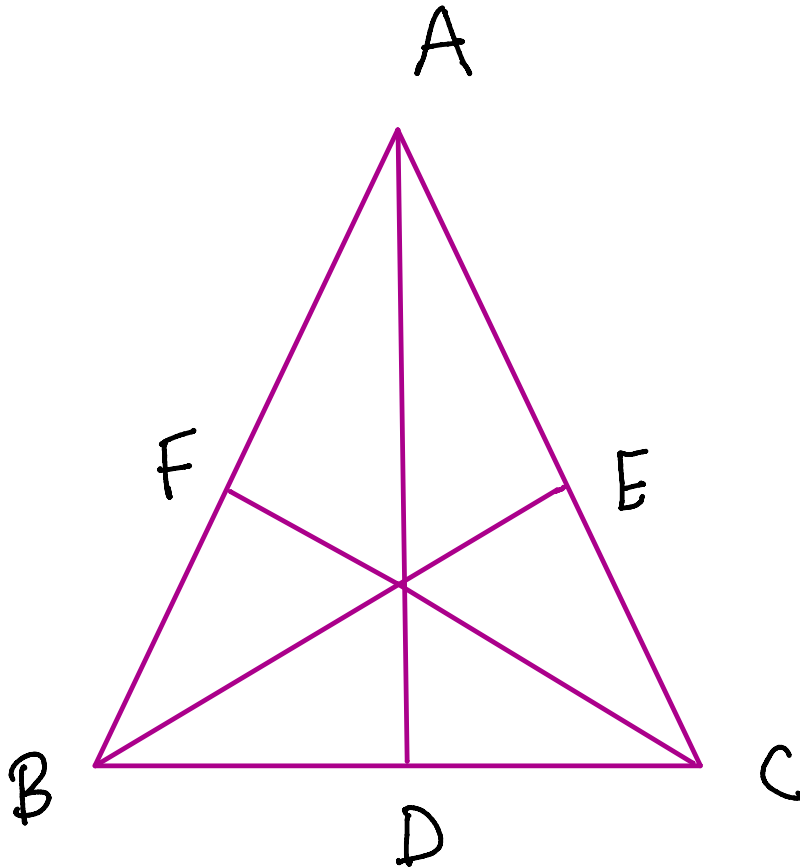
(a) $1/2$

(b) $7/8$

(c) $5/6$

(d) $3/4$

Solution.



By Apollonius Theorem, the sum of the squares of the two sides of a triangle is equal to twice the square of half the 3rd side plus twice the square of the median which bisects the 3rd side.

∴ We can write,

$$\begin{aligned} AB^2 + AC^2 &= 2(AD^2 + BD^2) \\ \Rightarrow &= 2AD^2 + \frac{BC^2}{2} \end{aligned}$$

$$\Rightarrow 2(AB^2 + AC^2) = BC^2 + 4AD^2 \quad \text{--- (1)}$$

Similarly for medians BE and CF,

$$2(BC^2 + AB^2) = AC^2 + 4BE^2 \quad \text{--- (2)}$$

$$\text{and} \quad 2(AC^2 + BC^2) = AB^2 + 4CF^2 \quad \text{--- (3)}$$

Now, adding equation (1), (2) and (3),

$$3(AB^2 + BC^2 + AC^2) = 4AD^2 + BE^2 + CF^2$$

$$\Rightarrow \frac{AD^2 + BE^2 + CF^2}{AB^2 + BC^2 + AC^2} = \frac{3}{4}$$

Ans.
Option(d).