

## Apollonius Theorem

This is an important theorem used in the questions related to triangles.

It states that,

"The sum of the squares of the two sides of a triangle is equal to twice the square of half the 3<sup>rd</sup> side plus twice the square of the median which bisects the 3<sup>rd</sup> side."

Proof.

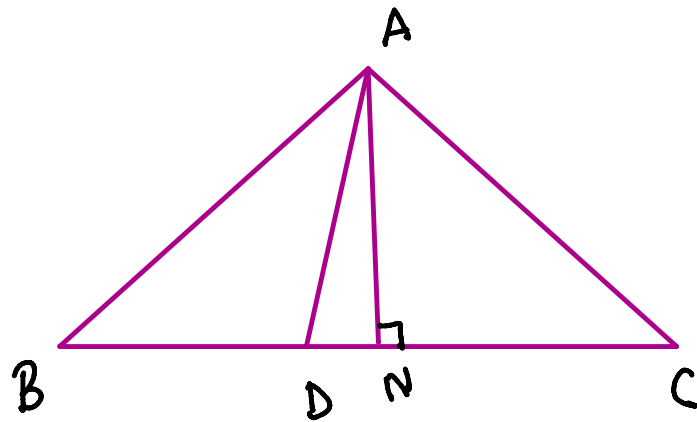
Let a  $\triangle ABC$  with  $AD$  as median and  $AN$  as altitude to  $BC$ .

In  $\triangle ABN$ ,

$$AB^2 = AN^2 + BN^2 \quad \text{--- (1)}$$

In  $\triangle ANC$ ,

$$AC^2 = AN^2 + NC^2 \quad \text{--- (2)}$$



Adding equations (1) and (2):  $\rightarrow$

$$\begin{aligned}AB^2 + AC^2 &= AN^2 + BN^2 + AN^2 + NC^2 \\&= 2AN^2 + (DC - DN)^2 + BN^2 \\&= 2AN^2 + (BD + DN)^2 + (DC - DN)^2 \\&= 2AN^2 + BD^2 + DN^2 + 2BD \cdot DN \\&\quad + DC^2 + DN^2 - 2DC \cdot DN \\&= 2(AN^2 + DN^2) + BD^2 + CD^2 \\&\quad + 2DC \cdot DN - 2DC \cdot DN \\&\quad [\because BD = DC] \\&= 2AD^2 + BD^2 + BD^2 \\&= 2AD^2 + 2BD^2\end{aligned}$$

$\Rightarrow$

$$AB^2 + AC^2 = 2 \left\{ \frac{BC}{2} \right\}^2 + 2AD^2$$

Proved