Q. $\frac{d^2x}{dy^2} = ?$ (In terms of $\frac{dy^2}{dx^2}$)

$$\frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\frac{1}{dy/dx}$$

$$= \frac{d}{dx}\left(\frac{1}{dy/dx}\right) \times \frac{dx}{dy}$$

$$= -\frac{d^2y/dx^2}{(dy/dx)^2} \times \frac{dx}{dy}$$

$$= \frac{-\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3}$$

Q. If $y = x + e^x$ then $\frac{d^2x}{dy^2} = ?$

$$\frac{dy}{dx} = 1 + e^x \qquad \frac{d^2y}{dx^2} = e^x$$

$$\frac{d^2x}{dy^2} = -\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{dy}{dx}\right)^3} = \frac{-e^x}{(x+e^x)^3}$$

Q. If $\frac{d}{dx}f(x) = g(x)$ & $\frac{d}{dx}g(x) = f(x^2)$

then, $\frac{d^2}{dx^2}(f(x^3)) = kx^a \cdot f(x^b) + px\, g(x^c)$

find k, a, b, c, p?

$$\frac{d^2}{dx^2}(f(x^3)) = \frac{d}{dx}\left(\frac{d}{dx}f(x^3)\right)$$

$$= \frac{d}{dx}\left(f'(x^3)\, 3x^2\right)$$

$\left[\because f''(x) = f(x^2)\right]$

$= 3x^2 f''(x^3)\, 3x^2 + f'(x^3)\, 6x$

$= 9x^4 f''(x^3) + 6x f'(x^3)$

$= 9x^4 \cdot f''(x^3) + 6x\, g(x^3)$

$= 9x^4 \cdot f(x^6) + 6x\, g(x^3)$

$k = 9,\ a = 4,\ b = 6,\ p = 6,\ c = 3$

** ~~$y_1\ y_2$~~ **Based Qs** (y_1 means $\frac{dy}{dx}$, y_n means $\frac{d^n y}{dx^n}$)

Q. If $y = (\sin^{-1} x)^2$ make differential eqn. in terms of y_2?

$y_1 = \frac{2\sin^{-1}x}{\sqrt{1-x^2}}$

$y_2 = \frac{\sqrt{1-x^2} \times \frac{2}{\sqrt{1-x^2}} - 2\sin^{-1}x \times \frac{-2x}{(1-x^2)^{1/2}}}{1-x^2}$

$y_2 = \frac{2(1-x^2)^{1/2} + 4x\sin^{-1}x}{(1-x^2)^{3/2}}$

$y_2 = \frac{2(1-x^2)^{1/2} + 2x\, y_1\sqrt{1-x^2}}{(1-x^2)^{3/2}}$

$y_2 = \frac{2(1-x^2) + 2xy_1 \sin^{-1}x}{(1-x^2)^2}$

$y_2(1-x^2) = 2 + 2xy_1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$y_2(1-x^2) = 2 + x(y_1)^2$

$(1-x^2)\, y_2 - x(y_1)^2 = 2$

$\sqrt{1-x^2} \cdot y_1 = 2\sin^{-1}x$

$y_2\sqrt{1-x^2} + \frac{y_1 \cdot -x}{\sqrt{1-x^2}} = \frac{2}{\sqrt{1-x^2}}$

$\underline{\underline{(1-x^2)y_2 - xy_1 = 2}}$

Q. If $y = e^{\tan^{-1}x}$ then P.T $(1+x^2)^2 y_2 + 2xy_1(1+x^2) = y$

$y_1 = \frac{e^{\tan^{-1}x}}{1+x^2}$

$y_1 = \frac{y}{1+x^2}$

$$y_2 = \frac{(1+x^2)y_1 - y(2x)}{(1+x^2)^2}$$

$$(1+x^2)^2 y_2 + 2xy = (1+x^2)y_1$$

$$(1+x^2)^2 y_2 + 2xy_1(1+x^2) = (1+x^2)y_1 - 2xy + 2xy_1$$

$$= y - 2xy + 2xy_1(1+x^2)$$

$$= y - \cancel{2xy} + \cancel{2xy}$$

$$(1+x^2)^2 y_2 + 2xy_1(1+x^2) = y$$

Q. $x^2+y^2=1$ then $y \cdot y'' + (y')^2$

$$2x + 2yy_1 = 0$$

$$x + yy' = 0$$

$$1 + yy'' + (y')^2 = 0$$

$$yy'' + (y')^2 = -1$$

Infinite Series

Mntra – writing y wherever first element recieved!

Q $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \ldots \infty}}}$

$$y = \sqrt{\sin x + y}$$

$$y^2 = \sin x + y$$

$$y^2 - y = \sin x$$

$$2y\frac{dy}{dx} - \frac{dy}{dx} = \cos x$$

$$\frac{dy}{dx} = \frac{\cos x}{(2y-1)}$$

☆☆ If $y = \sqrt{f(x) + \sqrt{f(x) + \ldots \infty}}$

then $\boxed{\frac{dy}{dx} = \frac{f'(x)}{2y-1}}$

Q $x = e^{y + e^{y + e^{y + \ldots \infty}}}$

$$\boxed{\begin{aligned} \cancel{\frac{dx}{dy}} &= \cancel{\frac{e^y}{2x-1}} \\ \frac{dy}{dx} &\cancel{=} \cancel{\frac{2x-1}{e^y}} \end{aligned}}$$

$x = e^{y+x}$

$\ln x = x + y$

$\frac{1}{x} = 1 + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1-x}{x}$

Q $y = \frac{x}{a+} \frac{x}{b+} \frac{x}{a+} \frac{x}{b+} \ldots \infty$, $\frac{dy}{dx} = ?$

$$y = \cfrac{x}{a + \cfrac{x}{b + \cfrac{x}{a + x \ldots \infty}}}$$

$$y = \cfrac{x}{a + \cfrac{x}{b+y}} = \frac{x(b+y)}{(ab + ay + x)}$$

$aby + ay^2 + \cancel{yx} = bx + \cancel{xy}$

$aby + ay^2 = bx$

$ab\frac{dy}{dx} + 2ya\frac{dy}{dx} = b$

$\frac{dy}{dx} = \frac{b}{ab + 2ay}$

Diff^n of Determinants

If determinant has variable terms then if differentiation is possible.

$$D(x) = \begin{vmatrix} f(x) & g(x) & h(x) \\ p(x) & q(x) & r(x) \\ u(x) & v(x) & \omega(x) \end{vmatrix}$$

$$D'(x) = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ p(x) & q(x) & r(x) \\ u & v & \omega \end{vmatrix} + \begin{vmatrix} f & g & h \\ p' & q' & r' \\ u & v & \omega \end{vmatrix} + \begin{vmatrix} f & g & h \\ p & q & r \\ u' & v' & \omega' \end{vmatrix}$$

Q. $f(x) = \begin{vmatrix} \cos(x+x^2) & \sin(x+x^2) & -\cos(x+x^2) \\ \sin(x-x^2) & \cos(x-x^2) & \sin(x-x^2) \\ \sin 2x & 0 & \sin 2x^2 \end{vmatrix}$

find $f'(x)$?

As fxn is complex so, it is preferable to solve det first.

$A = x + x^2$, $B = x - x^2$

$A + B = 2x$ $\quad$ $A - B = 2x^2$

$$f(x) = \begin{vmatrix} \cos A & \sin A & -\cos A \\ \sin B & \cos B & \sin B \\ \sin(A+B) & 0 & \sin(A-B) \end{vmatrix}$$

Expanding along R_3

$$f(x) = \sin(A+B)\cos(A-B) + \sin(A-B)\cos(A+B)$$
$$= \sin(A+B+A-B)$$
$$= \sin 2A$$
$$f(x) = \sin(2x+2x^2)$$
$$f'(x) = \cos(2x+2x^2) \cdot (2+4x)$$

Q. $$f(x) = \begin{vmatrix} 3 & 6x^2 & 1 \\ 2 & 2x^3 & a \\ 1 & x^4 & a^2 \end{vmatrix} \quad f''(a) = ?$$

$$f''(x) = \begin{vmatrix} 3 & 12 & 1 \\ 2 & 12a & a \\ 1 & 12a^2 & a^2 \end{vmatrix}$$

$$f''(a) = \begin{vmatrix} 3 & 12 & 1 \\ 2 & 12a & a \\ 1 & 12a^2 & a^2 \end{vmatrix} = 0$$

Q Let $\begin{vmatrix} x & 2 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$

① $E = ?$ Put $x = 0$, $E = 0$

② $D = ?$ diff. both sides & putting $x = 0$

$$D = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = -1$$

③ $A+B+C+D+E = ?$ Put $x = 1$

$A+B+C+D+E = 0$

④ $4A + 3B + 2C + 2D$

diff. & putting $x = 1$

$$\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 4A + 3B + 2C + 2D = 1$$

⑤ $A + B + C + 2E$

$= A + B + C + D + E \quad - D + E$

$= \quad 0 + 1 = 0 = 1$

8. $f(x) = -4 . e^{\frac{1-x}{2}} + \frac{x^3}{3} + \frac{x^2}{2} + x + 1$

& $g(x)$ being inverse of $f(x)$ &

$h(x) = \frac{a + bx^{3/2}}{x^{5/4}}$, $h'(5) = 0$ then

$\frac{a^2}{5b^2 \, g'(-7/6)} = ?$

$$g'(f(x)) = \frac{1}{f'(x)}$$

$g'(f(1)) = \frac{1}{f'(1)}$ | $f'(x) = 2e^{\frac{1-x}{2}} + x^2 + x^2 + 1$

$f'(1) = 2 + 1 + 1 + 1 = 5$

$g'(-7/6) = \frac{1}{5}$

$5g'(-7/6) = 1$

$h(x) = a\, x^{-5/4} + b\, x^{1/4}$

$h'(x) = -\frac{5}{4} a\, x^{-9/4} + \frac{1}{4} b x^{-3/4}$

$$h'(5) = -\frac{5}{4} a (5)^{-9/4} + \frac{1}{4} b (5)^{-3/4}$$

$$0 = -\frac{5}{4} a (5)^{-9/4} + \frac{1}{4} (b)(5)^{-3/4}$$

$$\frac{5}{\cancel{4}} a (5)^{-9/4} = \frac{1}{\cancel{4}} (b)(5)^{-3/4}$$

$$5a = b\, 5^{3/2}$$

$$a = b\, 5^{1/2}$$

$$\frac{a^2}{b^2} = 5$$

$$\frac{a^2}{5b^2} \quad g'(-7/6) = 5$$

Q. If $f(x)$ is a diff. fxn such that $f(2x) =$ ~~such that~~ $f'(x) f''(x)$

A) Deg. of polynomial

B) Polynomial itself.

$$f(2x) = f'(x) f''(x)$$

A) Let, degree of fxn be n.

n degree

$$n = (n-1) + (n-2)$$

$$n = 2n - 3$$

$\underline{n = 3}$ degree of poly $= 3$

B) Let, $f(x) = ax^3 + bx^2 + cx + d$

$$8ax^3 + 4bx^2 + 2cx + d = (3ax^2 + 2bx + c)(6ax + 2b)$$

$$8ax^3 + 4bx^2 + 2cx + d = 18a^2x^3 + (6ab + 12b)x^2 + (4b^2 + 6ac)x + 2bc$$

$8a = 18a^2$ $\qquad$ $6ab + 12b = 4b$

$a = 0, 9/4$ $\qquad$ $6ab = 8b$

$b = 0, 27/16$

$2c = 4b^2 + 6ac$

$2c = 4b^2 + \overset{3}{6} \times \frac{9}{\cancel{4}_2} c$

Solve to find $\underline{a, b, c,} d.$

Q. If $f(x)$ is diff. fxn such that $f(x+2y) = f(x) + f(2y) + 6xy(x+2y)$ $\forall$ $x, y \in R$ then $f''(0)$ $f''(1)$, $f''(2)$ are in ?

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{f(x)} + f(2h) + 6xh(x+2h) - \cancel{f(x)}}{h}$$

$$= \lim_{h \to 0} f'(2h) + 6x^2 + 24xh$$

$f'(x) = 6x^2$

Put $x, y = 0$

$f(0) = f(0) + f(0) + 0$

$f(0) = 0$

$f''(x) = 12x$

$f''(0) = 0$, $f''(1) = 12$, $f''(2) = 24$

Q. A twice diff. fxn is defined for for all real number & satisfies $f(0) = 2$, $f'(0) = -5$, $f''(0) = 3$ & $g(x)$ is defined as $g(x) = e^{ax} + f(x)$ $\forall$ $x \in R$

if $g'(0) + g''(0) = 0$ then $a = ?$

$$g(0) = e^{a0} + f(0) = 1 + 2 = 3$$

$$g'(x) = ae^{ax} + f'(x), \quad g'(0) = a - 5$$

$$g''(x) = a^2 e^{ax} + f''(0)$$

$$g''(0) = a^2 + 3$$

$$g'(0) + g''(0) = 0$$

$$a - 5 + a^2 + 3 = 0$$

$$a + a^2 - 2 = 0$$

$$a^2 + a - 2 = 0$$

$$a = 1, -2$$