

For the four circles M, N, O and P, following four equations are given :

Circle M :  $x^2 + y^2 = 1$

Circle N :  $x^2 + y^2 - 2x = 0$

Circle O :  $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P :  $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further center of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

- A Rhombus
- B Square
- C Rectangle
- D Parallelogram

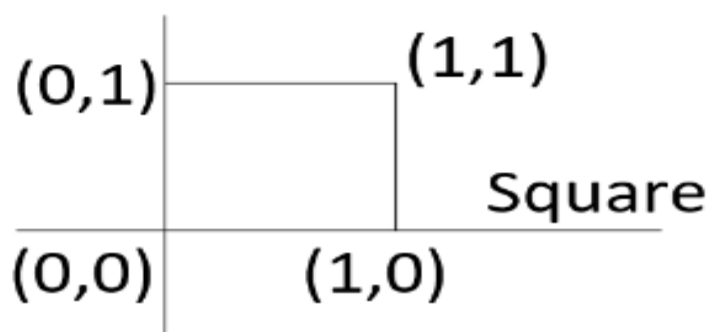
### Explanation

$$C_M = (0, 0)$$

$$C_N = (1, 0)$$

$$C_O = (1, 1)$$

$$C_P = (0, 1)$$



Choose the correct statement about two circles whose equations are given below :

$$x^2 + y^2 - 10x - 10y + 41 = 0$$

$$x^2 + y^2 - 22x - 10y + 137 = 0$$

- A circles have same centre
- B circles have no meeting point
- C circles have only one meeting point**
- D circles have two meeting points

### Explanation

Let  $S_1: x^2 + y^2 - 10x - 10y + 41 = 0$

$$\Rightarrow (x - 5)^2 + (y - 5)^2 = 9$$

Centre  $(C_1) = (5, 5)$

Radius  $r_1 = 3$

$S_2: x^2 + y^2 - 22x - 10y + 137 = 0$

$$\Rightarrow (x - 11)^2 + (y - 5)^2 = 9$$

Centre  $(C_2) = (11, 5)$

Radius  $r_2 = 3$

distance  $(C_1 C_2) = \sqrt{(5 - 11)^2 + (5 - 5)^2}$

distance  $(C_1 C_2) = 6$

$\therefore r_1 + r_2 = 3 + 3 = 6$

$\therefore$  circles touch externally

Hence, circle have only one meeting point.

Let the circle  $S : 36x^2 + 36y^2 - 108x + 120y + C = 0$  be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines,  $x - 2y = 4$  and  $2x - y = 5$  lies inside the circle  $S$ , then :

- A  $\frac{20}{9} < C < \frac{14}{3}$   
B  $100 < C < 165$   
C  $81 < C < 156$   
D  $100 < C < 156$

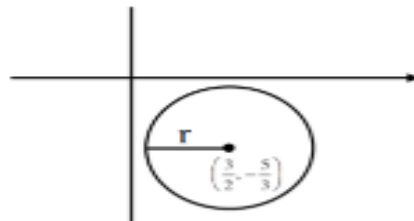
### Explanation

$$S : 36x^2 + 36y^2 - 108x + 120y + C = 0$$

$$\rightarrow x^2 + y^2 - 3x + \frac{10}{3}y + \frac{C}{36} = 0$$

$$\text{Centre} = (-g, -f) = \left(\frac{3}{2}, -\frac{5}{3}\right)$$

$$\text{radius} = r = \sqrt{\frac{9}{4} + \frac{25}{9} - \frac{C}{36}}$$



Now,

$$\rightarrow r < \frac{3}{2}$$

$$\rightarrow \frac{9}{4} + \frac{25}{9} - \frac{C}{36} < \frac{9}{4}$$

$$\rightarrow C > 100 \dots\dots (1)$$

Now, point of intersection of  $x - 2y = 4$  and  $2x - y = 5$  is  $(2, -1)$ , which lies inside the circle  $S$ .

$$\therefore S(2, -1) < 0$$

$$\rightarrow (2)^2 + (-1)^2 - 3(2) + \frac{10}{3}(-1) + \frac{C}{36} < 0$$

$$\rightarrow 4 + 1 - 6 - \frac{10}{3} + \frac{C}{36} < 0$$

$$C < 156 \dots\dots (2)$$

From (1) & (2)

$$100 < C < 156 \text{ Ans.}$$