Practice Questions

Q1. Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, -9). Find its centre and radius.

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Q2. The equation of the circle having centre (1, -2) and passing through the point of intersection of the lines $3x + y = 14$ and $2x + 5y = 18$ is

1. $x^2 + y^2 - 2x + 4y - 20 = 0$
2. $x^2 + y^2 - 2x - 4y - 20 = 0$
3. $x^2 + y^2 + 2x - 4y - 20 = 0$
4. $x^2 + y^2 + 2x + 4y - 20 = 0$

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Q3. Find the equation of the circle which touches the both axes in first quadrant and whose radius is a.

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Q4. If a circle passes through the point (0, 0) (a, 0), (0, b) then find the coordinates of its centre.

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Solution and Hints

S1. This is a simple application of class notes formulas. If one remebers the formulas using some trick, then it can be done easily just by putting values in formulas. We can solve 3 equations to get 3 unknowns of general form. By substitution of coordinates in the general equation of the circle given by

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

, we have

$$\begin{aligned} 40g + 6f + c &= -409 \\ 38g + 16f + c &= -425 \\ 4g - 18f + c &= -85 \end{aligned}$$

From these three equations, we get g = - 7, f = - 3 and c = -111 Hence, the equation of the circle is

$$\begin{aligned} & x^2 + y^2 - 14x - 6y - 111 = 0 \\ \implies & (x-7)^2 + (y-3)^2 = 132 \end{aligned}$$

Therefore, the centre of the circle is (7, 3) and radius is 13.

S2. Note that to get intersection point of two lines we just need to solve two linear equation problem.

$$\begin{aligned} 3x + y - 14 &= 0 \\ 2x + 5y - 18 &= 0 \end{aligned}$$

This gives us intersection point: $x = 4, y = 2$. Now radius of circle is:

$$\begin{aligned} r^2 &= (4-1)^2 + (2+2)^2 \\ &= 9 + 16 = 25 \end{aligned}$$

We have radius = 5. Then c in general form is,

$$\begin{aligned} c &= g^2 + f^2 - r^2 \\ &= (-1)^2 + (2)^2 - 25 = -20 \end{aligned}$$

So general form is,

$$x^2 + y^2 - 2x + 4y - 20 = 0$$

Hence option (1) is right.

S3. It can be done just by looking at the below picture,

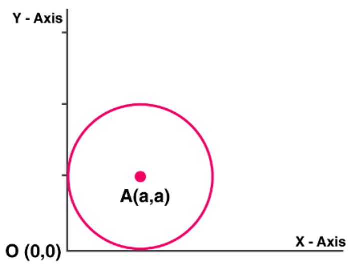

Figure 1: Circle with radius a

Now we can write equation of the center,

$$(x-a)^2 + (y-a)^2 = a^2$$

.

S4. This is also a simple application of class notes formulas, like que-1. By substitution of coordinates in the general equation of the circle given by

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

, we have

$$\begin{aligned} c &= 0 \\ a^2 + 2ga + c &= 0 \\ b^2 + 2fb + c &= 0 \end{aligned}$$

From these three equations, we get center as

$$\begin{aligned} -g &= \frac{a}{2} \\ -f &= \frac{b}{2} \end{aligned}$$

And radius to be,

$$r^2 = g^2 + f^2 - c = \frac{a^2 + b^2}{4}$$

Hence, the equation of the circle is

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$