Notes

Superposition principle:

When two or waves intersect at a point, the resultant displacement of that point is given by the vector sum of displacements due to each wave.

Reflection of wave:

When a wave is reflected by the boundary of a medium −

(i) its direction is reversed

(ii) if it is reflected by a denser medium then the wave suffers a phase change of π

So if $y_{incident} = A\sin(kx - wt)$ then $y_{reflected} = A\sin(kx + wt + \phi)$

$\phi = 0$ if the wave is reflected by a rarer medium and $\phi = \pi$ if reflected by denser medium

Standing waves:

When two waves of same frequency & same wavelength moving in opposite direction superimpose then the resultant wave is called a standing wave or stationary wave.

Equation of standing wave:

Let waves $y1 = A\sin(kx - wt)$ and $y2 = A\cos(kx + wt + \phi)$, then

$$yres = A\left(2\sin\left(\frac{kx - wt + kx + wt + \phi}{2}\right)\cos\left(\frac{kx - wt - kx - wt - \phi}{2}\right)\right)$$

$$yres = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(wt + \frac{\phi}{2}\right)$$

So we can see that the resultant wave is not a propagating wave (it is not a function of $kx \pm wt$), it is just oscillations going on at different points.

Also at positions where $kx + \frac{\phi}{2} = n\pi$, the ampitude of oscillation at those point is 0, such a point is called a node. Similarly at points where $kx + \frac{\phi}{2} = \frac{(2n+1)\pi}{2}$, the amplitude of oscillation is maximum, such a point is called an antinode.

Standing waves on a string tied at both ends:

Consider a string of Length l and having tension T and mass per unit length μ

$Speed\ of\ wave\ in\ this\ string\ is\ given\ by\ v = \sqrt{\frac{T}{\mu}}$

$For\ this\ string\ to\ vibrate\ in\ nth\ harmonic, the\ frequency\ of\ standing\ wave\ is\ given\ by$

$$f = \frac{nv}{2l}$$

$In\ this\ case\ there\ are\ n + 1\ nodes\ (including\ both\ ends), and\ n - 1\ antinodes$

$for\ n = 1\ the\ frequency\ is\ called\ the\ fundamental\ frequency.$

<u>$Standing\ waves\ on\ a\ string\ tied\ at\ one\ end$</u>:

$For\ this\ string\ to\ vibrate\ in\ (2n - 1)th\ harmonic, the\ frequency\ of\ wave\ is\ given\ by$

$$f = \frac{(2n - 1)v}{4l}$$

$In\ this\ case\ there\ are\ n\ nodes\ , and\ n\ antinodes$

$for\ n = 1\ the\ frequency\ is\ called\ the\ fundamental\ frequency.$