<table>
<tr><td>**8**</td><td># APPLICATION OF THE INTEGRALS</td></tr>
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KEY CONCEPT INVOLVED

Area Under Simple Curves

1. Let us find the area bounded by the curve $y = f(x)$, x-axis and the ordinated $x = a$ and $x = b$. Consider the area under the curve as composed of large number of thin vertical strips let there be an arbitary strip of hieght y and width dx. Area of elementary strip $dA = ydx$, where $y = f(x)$. Total Area A of the region between x-axis. ordinated $x = a$, $x = b$ and the curve $y = f(x)$ = Sum of areas of elementry thin strips across the region PQML
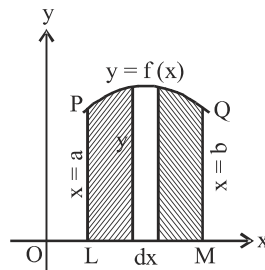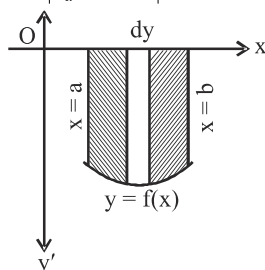

$$A = \int_a^b dA = \int_a^b ydx = \int_a^b f(x)dx$$

2. The area A of the region bounded by the curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d xdy$$
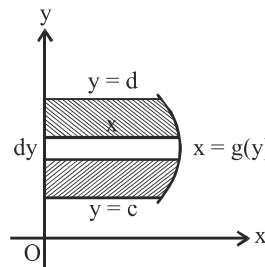
3. If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$, and the ordinates $x = a$, $x = b$ and x-axis is negative. But the numerical value of the area is to be taken into consideration. Then Area = $\left| \int_a^b f(x)dx \right|$

4. Let some portion of the curve is above x-axis and some portion is below x-axis. Let A_1 be the area below x-axis and A_2 be the area above of x-axis. Therefore Area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$.

$$A = |A_1| + A_2$$
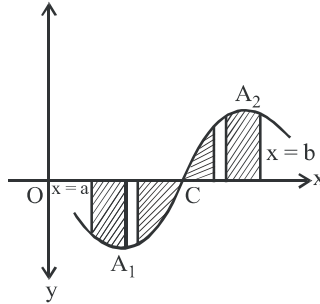
Area between Two curves

5. Let the two curves be $y = f(x)$ and $y = g(x)$. Suppose these curves intersect at $x = a$ and $x = b$. Consider the elementary strip of height y where $y = f(x) - g(x)$ with width dx
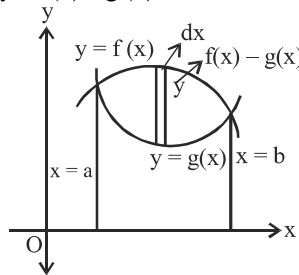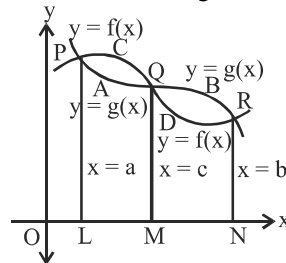

$\therefore \quad da = y\,dx$

$$\Rightarrow \quad A = \int_a^b \big(f(x) - g(x)\big)\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

i.e. A = Area bounded by the curve $y = f(x)$ − Area bounded by the curve $y = g(x)$

6. If the two curves $y = f(x)$ and $y = g(x)$ intersects at $x = a$, $x = c$ and $x = b$ such that $a < c < b$. If $f(x) > g(x)$ in [a, c] and $f(x) < g(x)$ in [c, b], Then the area of the regions bounded by curve.

$$= \text{Area of the region PAQCP} + \text{Area of the region QDRBQ} = \int_a^c \big(f(x) - g(x)\big)\,dx + \int_c^b \big(g(x) - f(x)\big)\,dx$$