

Question 1: The general solution of $\sin x - 3 \sin 2x + \sin 3x = \cos x - 3 \cos 2x + \cos 3x$ is _____.

Solution:

$$\begin{aligned} \sin x - 3 \sin 2x + \sin 3x &= \cos x - 3 \cos 2x + \cos 3x \\ \Rightarrow 2 \sin 2x \cos x - 3 \sin 2x - 2 \cos 2x \cos x + 3 \cos 2x &= 0 \\ \Rightarrow \sin 2x (2 \cos x - 3) - \cos 2x (2 \cos x - 3) &= 0 \\ \Rightarrow (\sin 2x - \cos 2x) (2 \cos x - 3) &= 0 \\ \Rightarrow \sin 2x &= \cos 2x \\ \Rightarrow 2x &= 2n\pi \pm (\pi / 2 - 2x) \text{ i.e.,} \\ x &= n\pi / 2 + \pi / 8 \end{aligned}$$

Question 2: If $\sec 4\theta - \sec 2\theta = 2$, then the general value of θ is _____.

Solution:

$$\begin{aligned} \sec 4\theta - \sec 2\theta = 2 &\Rightarrow \cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta \\ \Rightarrow -\cos 4\theta &= \cos 6\theta \\ \Rightarrow 2 \cos 5\theta \cos \theta &= 0 \\ \Rightarrow H &= [h \cot 15^\circ] / [\cot 15^\circ - 1] \text{ or} \\ n\pi/5 + \pi/10 \end{aligned}$$

Question 3: If $\tan(\cot x) = \cot(\tan x)$, then $\sin 2x =$ _____.

Solution:

$$\begin{aligned} \tan(\cot x) = \cot(\tan x) &\Rightarrow \tan(\cot x) = \tan(\pi / 2 - \tan x) \\ \cot x &= n\pi + \pi / 2 - \tan x \\ \Rightarrow \cot x + \tan x &= n\pi + \pi / 2 \\ 2 \sin 2x &= n\pi + \pi / 2 \\ \Rightarrow \sin 2x &= 2 / [n\pi + \pi / 2] \\ &= 4 / \{(2n + 1) \pi\} \end{aligned}$$

Question 4: If the solution for θ of $\cos p\theta + \cos q\theta = 0$, $p > 0$, $q > 0$ are in A.P., then numerically the smallest common difference of A.P. is _____.

Solution:

$$\begin{aligned} \text{Given } \cos p\theta &= -\cos q\theta = \cos(\pi + q\theta) \\ p\theta &= 2n\pi \pm (\pi + q\theta), n \in \mathbb{I} \\ \theta &= [(2n + 1)\pi] / [p - q] \text{ or } [(2n - 1)\pi] / [p + q], n \in \mathbb{I} \end{aligned}$$

Both the solutions form an A.P. $\theta = [(2n + 1)\pi] / [p - q]$ gives us an A.P. with common difference $2\pi / [p - q]$ and $\theta = [(2n - 1)\pi] / [p + q]$ gives us an A.P. with common difference $= 2\pi / [p + q]$.

Certainly, $\{2\pi / [p + q]\} < \{2\pi / [p - q]\}$