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Notes -

Recap:

→ If ~~cosx~~ $\sin x = \sin y$, $x = n\pi + (-1)^n y$, $n \in \mathbb{Z}$

→ If $\cos x = \cos y$, $x = 2n\pi \pm y$, $n \in \mathbb{Z}$

Q→ $\cos x = \frac{-1}{2}$

So→ $\cos x = \cos \frac{2\pi}{3}$

$\Rightarrow x = 2n\pi \pm \frac{2\pi}{3}$

$n = 0 \Rightarrow x = \frac{2\pi}{3}, \frac{-2\pi}{3}$

$n = 1 \Rightarrow x = 2\pi + \frac{2\pi}{3}, 2\pi - \frac{2\pi}{3}$

$n = -1 \Rightarrow x = \frac{2\pi}{3} - 2\pi, -2\pi - \frac{2\pi}{3}$

→ If $\tan x = \tan y$, then
$x = n\pi + y$, $n \in \mathbb{Z}$

x & y are not odd multiples of $\frac{\pi}{2}$.

Q→ Find the general solution to
$a\cos\theta + b\sin\theta = c$ (a, b, c are real)

So→ $$\frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$$

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Let $\frac{a}{\sqrt{a^2+b^2}} = \cos\phi$

$\therefore \frac{b}{\sqrt{a^2+b^2}} = \sin\phi$

$\Rightarrow \cos\phi\cos\theta + \sin\phi\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$

$\Rightarrow \cos(\phi-\theta) = \frac{c}{\sqrt{a^2+b^2}} = \cos\gamma$ $\left(\text{let } \cos\gamma = \frac{c}{\sqrt{a^2+b^2}}\right)$

$\Rightarrow \cos(\theta-\phi) = \cos\gamma$

$\Rightarrow \cancel{\theta =}$

$\Rightarrow \cancel{\phi - \theta = 2\pi}$

$\Rightarrow \theta - \phi = 2n\pi \pm \gamma \quad \forall n \in \mathbb{Z}$

$\Rightarrow \theta = 2n\pi \pm \gamma + \phi$

Q. The solution set of the system of equations

$x+y = \frac{2\pi}{3}$

$\cos x + \cos y = \frac{3}{2}$

where x, y are real is ---

Ans

$y = \frac{2\pi}{3} - x$

$\Rightarrow \cos x + \cos\left(\frac{2\pi}{3} - x\right) = \frac{3}{2}$

$\Rightarrow \cos x + \cos\frac{2\pi}{3}\cos x + \sin\frac{2\pi}{3}\sin x = \frac{3}{2}$

$\Rightarrow \cos x - \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}$

$\Rightarrow \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{3}{2}$

$\Rightarrow \cos\frac{\pi}{3}\cos x + \sin\frac{\pi}{3}\sin x = \frac{3}{2}$

$\Rightarrow \cos\left(\cancel{\pi} x - \frac{\pi}{3}\right) = \frac{3}{2}$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos \gamma \quad \left(\text{Let } \frac{3}{2} = \cos\gamma\right)$

$\Rightarrow x - \frac{\pi}{3} = 2n\pi \pm \gamma$

$\Rightarrow x = 2n\pi + \frac{\pi}{3} \pm \gamma$

Is this the solⁿ?
Surprisingly no!!

Because $\cos\gamma \leq 1$ but $\frac{3}{2} \nleq 1$

$\therefore$ No solution exists

Q→ Find the general solⁿ of
$\tan^2\theta + \sec 2\theta = 1$

Sol→ $\tan^2\theta + \frac{1}{\cos^2\theta - \sin^2\theta} = 1$

$\Rightarrow \tan^2\theta + \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta} = 1$

$\Rightarrow \tan^2\theta + \frac{1 + \tan^2\theta}{1 - \tan^2\theta} = 1$

$\Rightarrow \tan^2\theta(1 - \tan^2\theta) + 1 + \tan^2\theta = 1 - \tan^2\theta$

$\Rightarrow 2\tan^2\theta + 1 - \tan^4\theta = 1 - \tan^2\theta$

$\Rightarrow \tan^2\theta(\tan^2\theta - 3) = 0$

$\Rightarrow \tan\theta = 0 \Rightarrow \boxed{\theta = n\pi}$

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$\Rightarrow \tan^2\theta = 3$

$\Rightarrow \tan\theta = \pm\sqrt{3}$

~~$\Rightarrow \tan\theta = \pm\frac{\pi}{3}$~~

$\Rightarrow \tan\theta = \tan\left(\pm\frac{\pi}{3}\right)$

$\Rightarrow \theta = n\pi \pm \frac{\pi}{3}, n \in Z$

$\therefore$ sol^n set is

$\{n\pi \mid n \in Z\} \cup \left\{n\pi + \frac{\pi}{3} \mid n \in Z\right\} \cup \left\{n\pi - \frac{\pi}{3} \mid n \in Z\right\}$