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Problems -

Q1) Find the values of $x \in (-\pi, \pi)$ which satisfy the equation
$$1 + |\cos x| + |\cos^2 x| + \dots = 4$$

Sol) $|\cos^n x| = (|\cos x|)^n$

$\Rightarrow 1 + |\cos x| + |\cos x|^2 + |\cos x|^3 + \dots$

$\Rightarrow 1 + c + c^2 + c^3 + \dots$ (where $c = |\cos x|$)

$\Rightarrow \dfrac{1}{1-c}$ iff $|c| < 1$ i.e. $|\cos x| < 1$ which is valid for all x ~~$\{0\}$~~ in $\{-\pi, \pi\} - \{0\}$

$\Rightarrow \dfrac{1}{1 - |\cos x|} = 4$

$\Rightarrow |\cos x| = \dfrac{3}{4}$ ~~$= \cos \gamma$~~

$\Rightarrow \cos x = \pm \dfrac{3}{4}$ ~~$= \cos$~~

$\Rightarrow \cos x = \dfrac{3}{4} = \cos \gamma$ & $\cos x = \dfrac{-3}{4} = \cos \beta$

$\Rightarrow x = 2n\pi \pm \gamma$ $\Rightarrow x = 2n\pi \pm \beta$

$\therefore \{2n\pi \pm \gamma \mid n \in \mathbb{Z}\} \cup \{2n\pi \pm \beta \mid n \in \mathbb{Z}\}$

where ~~$\gamma \ldots$~~ $\cos \gamma = \dfrac{3}{4}$ & $\cos \beta = \dfrac{-3}{4}$

Q2) Let m be an odd integer. If $\sin mx = \sum_{p=0}^{m} b_p \sin^p x$ for every x, then find the values of b_0 & b_1.

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Ans → $\sin mx = b_0 + b_1 \sin x + b_2 \sin^2 x + \dots + b_m \sin^m x$

$\Rightarrow b_0 = 0$

$\frac{\sin mx}{\sin x} = b_1 + b_2 \sin x + \dots + b_m \sin^{m-1} x$

$\lim_{x \to 0} \frac{\sin mx}{\sin x} = \lim_{x \to 0} (b_1 + b_2 \sin x + \dots)$

$\Rightarrow m = b_1$

Q③ ~~Solve $\tan^4 y - \tan^2 y = 6$, $0 \le y < 2\pi$~~

Q③ Solve $\cos^3\theta - 4\cos^2\theta - \frac{1}{2}\cos\theta + 2 = 0$

Ans③ On hit & trial, $\cos\theta = 4$ is a solⁿ.

∴ Dividing $\cos^3\theta - 4\cos^2\theta - \frac{1}{2}\cos\theta + 2$, we get

$\cos^2\theta - \frac{1}{2} = 0$

$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$ & $\cos\theta = -\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4}$ & $\theta = 2n\pi \pm \frac{3\pi}{4}$

$\theta = \left\{2n\pi \pm \frac{\pi}{4} \mid n \in \mathbb{Z}\right\} \cup \left\{2n\pi \pm \frac{3\pi}{4} \mid n \in \mathbb{Z}\right\}$