NCERT EXEMPLAR SELECTED PROBLEMS : PROBLEM 6 ON ITF

36. The number of real solutions of the equation

$$\sqrt{1 + \cos 2x} = \sqrt{2} \cos^{-1} (\cos x) \text{ in } \left[\frac{\pi}{2}, \pi\right] \text{ is}$$

(a) 0   (b) 1   (c) 2   (d) Infinite

Sol. (a) We have, $\sqrt{1 + \cos 2x} = \sqrt{2} \cos^{-1}(\cos x)$, $x \in \left[\frac{\pi}{2}, \pi\right]$

$\Rightarrow \qquad \sqrt{2 \cos^2 x} = \sqrt{2} \cos^{-1} (\cos x)$

$\Rightarrow \qquad \sqrt{2} \cos x = \sqrt{2} \cos^{-1} (\cos x)$

$\Rightarrow \qquad \cos x = \cos^{-1} (\cos x)$

$\Rightarrow \qquad \cos x = x \qquad [\because \cos^{-1} (\cos x) = x]$

For $\quad x \in \left[\frac{\pi}{2}, \pi\right]$, $\cos x \le 0$

$\therefore \cos x = x$ is not possible for any value of x