NCERT EXEMPLAR SELECTED PROBLEMS : PROBLEM 5 ON ITF

35. If $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$, then $\alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$ equals

(a) 0   (b) 1   (c) 6   (d) 12

Sol. (c) We have, $\cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$

We know that, $0 \le \cos^{-1}x \le \pi$

$\Rightarrow \cos^{-1}\alpha + \cos^{-1}\beta + \cos^{-1}\gamma = 3\pi$ is possible if and only if

$$\cos^{-1}\alpha = \cos^{-1}\beta = \cos^{-1}\gamma = \pi$$

$$\Rightarrow \qquad \alpha = \beta = \gamma = -1$$

$$\therefore \qquad \alpha(\beta + \gamma) + \beta(\gamma + \alpha) + \gamma(\alpha + \beta)$$

$$= -1(-1-1) - 1(-1-1) - 1(-1-1) = 6$$