

NCERT EXEMPLAR SELECTED PROBLEMS :
PROBLEM 2 ON ITF

17. Find the value of $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$.

Sol. $4 \tan^{-1} \frac{1}{5} - \tan^{-1} \frac{1}{239}$

$$= 2 \left(2 \tan^{-1} \frac{1}{5} \right) - \tan^{-1} \frac{1}{239}$$
$$= 2 \tan^{-1} \frac{\frac{2}{5}}{1 - \left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \right)$$
$$= 2 \tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239}$$
$$= 2 \tan^{-1} \frac{5}{12} - \tan^{-1} \frac{1}{239}$$
$$= 2 \tan^{-1} \frac{\frac{2}{5}}{1 - \left(\frac{1}{5}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \right)$$
$$= 2 \tan^{-1} \frac{2/5}{24/25} - \tan^{-1} \frac{1}{239}$$
$$= 2 \tan^{-1} \frac{5}{12} - \tan^{-1} \frac{1}{239}$$
$$= \tan^{-1} \frac{2 \cdot \frac{5}{12}}{1 - \left(\frac{5}{12}\right)^2} - \tan^{-1} \frac{1}{239} \quad \left(\because 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2} \right)$$
$$= \tan^{-1} \frac{144 \times 5}{119 \times 6} - \tan^{-1} \frac{1}{239}$$

$$\begin{aligned}
&= \tan^{-1} \frac{120}{119} - \tan^{-1} \frac{1}{239} \\
&= \tan^{-1} \frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \cdot \frac{1}{239}} \quad \left(\because \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x - y}{1 + xy} \right) \\
&= \tan^{-1} \frac{120 \times 239 - 119}{119 \times 239 + 120} = \tan^{-1} \frac{28680 - 119}{28441 + 120} \\
&= \tan^{-1} \frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}
\end{aligned}$$