NCERT EXEMPLAR SELECTED PROBLEMS : PROBLEM 1 ON ITF

1. Find the value of $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$.

Sol. We know that, $\tan^{-1}\tan x = x, x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

So, $\tan^{-1}\left(\tan\frac{5\pi}{6}\right) \neq \frac{5\pi}{6}$ as $\frac{5\pi}{6} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Also $\cos^{-1}\cos x = x; x \in [0, \pi]$

So, $\cos^{-1}\left(\cos\frac{13\pi}{6}\right) \neq \frac{13\pi}{6}$ as $\frac{13\pi}{6} \notin [0, \pi]$

$\therefore$ $\tan^{-1}\left(\tan\frac{5\pi}{6}\right)+\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$$= \tan^{-1}\left[\tan\left(\pi-\frac{\pi}{6}\right)\right]+\cos^{-1}\left[\cos\left(2\pi+\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left(-\tan\frac{\pi}{6}\right)+\cos^{-1}\left(-\cos\frac{7\pi}{6}\right)$$

$$= -\tan^{-1}\left(\tan\left(-\frac{\pi}{6}\right)\right)+\left[\cos^{-1}\cos\left(\frac{\pi}{6}\right)\right] = -\frac{\pi}{6}+\frac{\pi}{6} = 0$$