

NCERT EXEMPLAR SELECTED PROBLEMS :

PROBLEM 10

Q29. Find the general solution of the equation $(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$.

Sol. $(\sqrt{3}-1)\cos\theta + (\sqrt{3}+1)\sin\theta = 2$

Put $\sqrt{3}-1 = r \sin\alpha$ and $\sqrt{3}+1 = r \cos\alpha$

$$\therefore r^2 = (\sqrt{3}-1)^2 + (\sqrt{3}+1)^2 \Rightarrow r^2 = 8 \Rightarrow r = 2\sqrt{2}$$

Also, $\tan\alpha = \frac{\sqrt{3}-1}{\sqrt{3}+1} = 2-\sqrt{3} \Rightarrow \alpha = \frac{\pi}{12}$

From eq. (i), we have

$$r \sin\alpha \cos\theta + r \cos\alpha \sin\theta = 2 \Rightarrow r \sin(\theta + \alpha) = 2$$

$$\Rightarrow \sin(\theta + \alpha) = \frac{1}{\sqrt{2}} \Rightarrow \sin(\theta + \alpha) = \sin \frac{\pi}{4}$$

$$\Rightarrow \theta + \alpha = n\pi + (-1)^n \frac{\pi}{4}, n \in Z$$

$$\Rightarrow \theta = n\pi + (-1)^n \frac{\pi}{4} - \frac{\pi}{12}, n \in Z$$